ALGORITHM FOR OPTIMAL CORRECTION OF TRAIN TRAFFIC SCHEDULE

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Abstract. The article analyses the problem of train traffic schedule optimal addition according to minimum overall expenditure criteria. The task is to analyze optimal insertion of an additional train track into already made schedule. The methodology is all-right applicable for ad hoc requests submission. Handling importance of ad hoc requests is emphasized in EU directive 2001/14/EB. The task can be solved applying Monte-Carlo, genetic algorithms, variants reselection methods or methods of the shortest (according to waste) way in graphs. All these methods are equally exact and can be implemented in modern computers with short enough time consumption.

Keywords: train traffic, additional train, optimization, overall expenditure, methodology, algorithm.

1. Introduction

Planning of train traffic is more rigorous than planning of road traffic. This occurs not only because trains are bulky, but because they can not move freely, changing their speed or overdriving each other as road transport facilities do.

Despite high requirements, deviations from railroad traffic schedule are likely to occur: trains fail behind the schedule because of various reasons. Depending on the category of delayed train and freight being transported by this train, traffic managers (dispatchers) correct the situation accepting appropriate heuristic decisions, which often are not optimal or sometimes even faulty.

Currently a high level of informational transport control systems presents a possibility to install automatic or even self-acting correction system for disturbed train traffic. Science fiction [1–4] contains similar tasks; however they are solved either by applying the criterion of passenger comfort or by adapting them for railroad infrastructure with semi-automatic blocking without possibility of operative correction for train mobility [2–4]. Such systems are not typical for railroad networks of various countries: semi-automatic blockings are already obsolete and modern traffic control technologies (e.g. the EU system ERTMS) enable to control train traffic at a high level of exactness.

This issue presents theoretical principles of optimal correction of train traffic schedule being disturbed in railroad line, having installed a self-acting control system.

The issue presents the algorithms, which might be successfully applied for automation of "last minute" (ad hoc) application service (by inserting an additional train line in a fully prepared traffic schedule). The current situation of "last minute" (ad hoc) application service is left as the option of infrastructure managers and the service procedure itself is such that the driver (applicant) is not aware of how to formulate the application and satisfy maximum of his interests as well as how to find out the detail, which may cancel the application [5, 6].

2. Formulation of the task

The train traffic schedule is comprised of a number of so called "train lines", each of them describes a particular route.

In the diagram the "train line" on the plane of time (t) and distance (s) is represented as a broken line, the horizontal segments of which correspond to standing (at the station) and oblique segments correspond to moving at appropriate speed.

Stations on the train schedule are labelled only by their axial line (i.e. the length of the station is not assessed). Acceleration and deceleration sections are

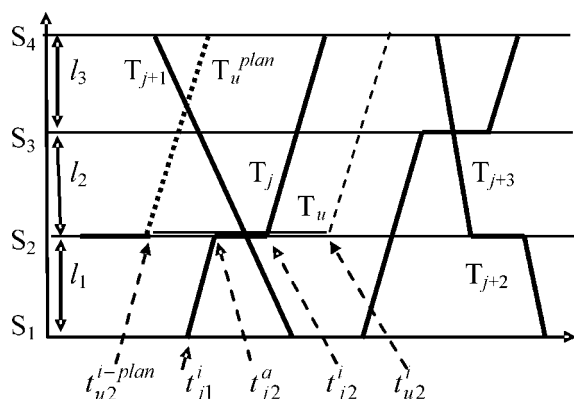not shown also: it seems that a train stops momentarily and in the same way develops necessary speed (Fig 1).

Fig 1. The example of a train traffic schedule diagram

The average speed T_j of train j in line i (the line, connecting stations S_i and S_{i+1}):

$$v_{ji} = \frac{t_{j(i+1)}^a - t_{ji}^i}{l_i} \quad (1)$$

is called technical speed of train T_j in line, connecting S_i and S_{i+1} stations.

In Fig 1, as well as in equations $t_{j(i+1)}^a$, the time when train T_j arrives to station S_{i+1} is presented.

Analytically "train line" of train j is described as function $s_j(t)$. In turn each function $s_j(t)$ is comprised of straights and segments:

$$s_j(t) = \begin{cases} a_{j1} + v_{j1}t & t_{j1}^i < t < t_{j2}^a, \\ a_{j2} & t_{j2}^a < t < t_{j2,}^i \\ a_{j2} + v_{j2}t & t_{j2}^i < t < t_{j3}^a, \\ \dots & \\ a_{j(n-1)} + v_{j(n-1)}t & t_{j(n-1)}^i < t < t_{jn}^a, \\ a_{jn} & t > t_{jn}^a. \end{cases} \quad (2)$$

If train j does not stop at station i, line $(i+1)$ of this function is expressed as:

$$a_{ji} \qquad t_{ji}^a = t_{ji}^i.$$

Herein: n – the number of station in line.
Traffic schedule:

$$S(t) = \{s_j(t)\}. \quad (3)$$

These parameters are enough to describe the traffic schedule nominally.

Departure time matrix $t^i = \left\| t_{ji}^i \right\|$.
Speed matrix $v = \left\| v_{ji} \right\|$.
Coordinate vector of station axial lines $S = \left\| S_i \right\|$.

The disturbance of traffic schedule is described as the difference between planned diagram $S^p(t)$ and factual diagram $S^f(t)$, as even for a single "train line" this equation is applied: $s_j^p(t) - s_j^f(t) > \Delta(s)$.

Herein $\Delta(s)$ is permissible delay of the train (measured in terms of losses).

Hereinafter such traffic disturbances are analyzed when only a single train fails behind the schedule impermissibly (Fig 1 train T_u).

Train T_u according to the plan had to run in line T_u^{plan} from station S_2 at the time moment t_{u2}^{i-plan}, however because of any reason T_u could not leave from station S_2 until time moment t_{u2}^i.

Losses incurred because of train T_u delay (compensations for clients' losses etc.) depend on the amount of delay time ($\tau_u = t_u^i - t_u^{i-plan}$) and the point of route (s) at which the train delays. These assumptions are expressed as function $N_u(\tau_u, s)$, for which these features are appropriate:

$$N_u(\tau_{1u}, s_1) > N_u(\tau_{2u}, s_2), \quad \text{as } \tau_{1u} > \tau_{2u}, \quad s_1 = s_2;$$

$$N_u(0, s) = 0.$$

Practically function $N_u(\tau_u, s)$ is discrete: s acquires coordinate values of station axial lines only $s \in \{s_z, ..., s_n\}$.

Herein: z is the station index, where traffic schedule of the train T_u was disturbed; n is the index of terminal station.

In such case $N_u(\tau_u, s) \rightarrow N_u(\tau_{ui})$.

Function $N_u(\tau_{ui})$ assesses the losses involved in delay of train T_u only. This is enough as the task of traffic optimization for delayed train is analyzed, but if a completely new train line must be inserted in a traffic schedule (i.e. as addition but not correction of traffic schedule) it is necessary to use the function:

$$W_{ui}^T = W_{Tui}^T + W_{Eui}^T + N_{ui}.$$

Herein: W_{Tui}^T – component depending on time and involved in exploitation costs of locomotives and carriages of train T_u as well as calculated for the period, during which the train T_u crosses line i ($S_i - S_{i+1}$);

W_{Eui}^T – Costs for the team of train T_u, calculated for the period, during which train T_u crosses line i;

N_{ui} – Losses, incurred because of ill-timed arrival of train T_u to terminal station S_{i+1} of line i ($S_i - S_{i+1}$).

If it were be possible to neglect other traffic participants, train T_u would run all the remaining distance (from station S_z, where it was forced to delay, up to terminal station S_n) at the reasonable speed

($v_u^{opt}(s) \leq v_{ui}^{rib}(s)$) resulted in minimal costs of fuel and losses W_u, incurred because of the delay:

$$W_u = \sum_{i=z}^{n} \left(W_{ui}^E + N_u(\tau_{ui}) \right) \rightarrow \min. \qquad (4)$$

In this formula:

$$W_{ui}^E = k_{ri} k_{ti} \left(G + \left(M + \frac{N}{Q_u} \right) w_{0ui} \right) \frac{Q_u l_i}{3440} \text{ [kWh], } (5)$$

G, M and N – parameters of locomotive, presented in its documentation;

Q_u – mass of train T_u, t;

l_i – the length of line i, km;

$k_{ri} = 1 + (0,705 - 0,00452 v_{ui}) i_e$ – coefficient of road profile;

v_{ui} – the train speed in line i, km/h,

i_e – equivalent inclination of the road, $^0/_{00}$;

$k_{ti} = 1 + \left(0,0022(t^0 + 15) - 0,0072 \right)(t^0 - 15)$ – coefficient of temperature;

t^0 – air temperature, $^{\circ}$C;

w_{0ui} – coefficient of train movement resistance.

$$w_{0ui} = 1,04 \frac{Q_L w_{0ui}^L + Q_V w_{0ui}^V}{Q_u = Q_L + Q_V}, \qquad (6)$$

$$w_{0ui}^L = 1,9 + 0,008 v_{ui} + 0,00025 v_{ui}^2,$$

$$w_{0ui}^V = 0,7 + \frac{3 + 0,09 v_{ui} + 0,002 v_{ui}^2}{17,5},$$

W_{ui}^E – expressions obtained in accordance with the methods introduced in reference [7].

The second component $N_u(\tau_{ui})$ of formula (4) has no universal expression and is stipulated by technological, arbitrary and other dependences. Seeking that all components in formula (4) would be of the same dimension, it is necessary to express $N_u(\tau_{ui})$ not in terms of money, but in terms of arbitrary fuel in kg (as W_{ui}^E). The fact permits an assumption that these dependences are known.

Unfortunately the assumption (made before the formula (4) has been written) that other traffic participants may be neglected (equal to the assumption that other traffic participants do not intervene) is often valid for road transport facilities, but not for trains.

In railroads with a modern traffic control system being implemented, train T_u, performing manipulations of its speed is able to:

1. To catch up with an oncoming slower train,

2. To be caught up by a faster train running after.

In the first case two events are possible again:

1.1. The ongoing train T_j stops at the primary station of the line, in which it will be caught up and lets train T_u pass.

1.2. The ongoing train does not stop until its panned station.
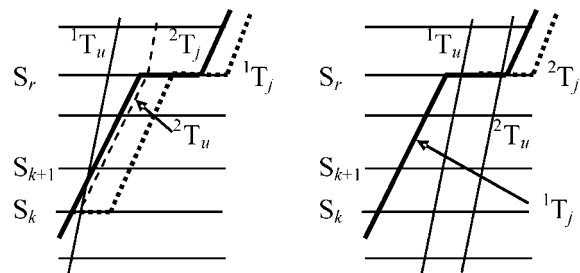
These cases are illustrated in Fig 2.

Fig 2. Traffic variations. On the left: slower train T_j makes way (the situation ^1T_u–^1T_j) or does not make way (the situation ^2T_u–^2T_j) for train T_u. On the right: train T_u does not disturb (the situation ^1T_u–^1T_j) or disturbs (the situation ^2T_u–^2T_j) train T_j to leave timely

In the second case train T_u has always to stop and let the train, which is running after, pass because faster trains are usually of higher category.

The analysis of variations of possible traffic situations shows that the situations below should be applied for the analysis of each line k (with its terminal station S_{k+1}):

A – train T_u neither in line k nor in its terminal station (S_{k+1}) does not catch up with ongoing train T_j and can not be caught up by train T_{j+1}, which is running after.

The other variation of this case: train T_j can not be caught up in line k, but according to schedule stops in the terminal station S_{k+1}, belonging to this line. In this case train T_u at station S_{k+1} is able to overtake train T_j, without disturbance of its traffic schedule (Right side of Fig 2 the situation ^1T_u–^1T_j).

B – train T_j can not be caught up in line k but according to schedule stops at terminal station S_{k+1}, of the line, where it is overtaken by train T_u, disturbing train T_j to leave timely from station S_{k+1} (Right side of Fig 2 the situation ^2T_u–^2T_j).

C – train T_u in line k as forecasted catches up with ongoing train T_j, which on purpose stops at the primary station S_k, of the line, avoiding traffic disturbance for train T_u. (Left side of Fig 2 the situation ^1T_u–^1T_j).

D – train T_u in line k catches up to ongoing train T_j, which does not let train T_u pass. The situation is complicated by the fact that according to the schedule T_j should not stop at terminal station S_{k+1} of the line.

E – train T_u in line k catches up with ongoing train T_j, which does not let train T_u to pass, but according to the schedule it has to stop at terminal station S_{k+1} of the line.

F – train T_u in line k is caught up by train T_{j+1}, which is running after and T_u is forced to let it pass, by stopping on purpose at primary station S_k of the line.

These entire situations are defined by different formulae of general costs.

Situation A. To run a single line k general costs are:

$$W_{uk} = W_{uk}^E + N_u(\tau_{uk}). \qquad (7a)$$

To run the whole distance from station S_z to terminal station S_n of the route general costs are:

$$W_{uy} = \sum_{i=z}^{n}(W_{ui}) + W_{uz}^S. \qquad (7b)$$

W_{ui} – alculated according to formula (7a), as $k \to i$; W_{uz}^S – the costs of arbitrary fuel for train T_u to develop speed at primary station S_z, are calculated according to the same methods as the costs of arbitrary fuel for train's speed development after its stopping (see formula (11)). W_{uk}^E and $N_u(\tau_{uk})$ are explained by commenting the formula (4).

Situation B.

$$W_{uy} = \sum_{i=z}^{n}(W_{ui}) + W_{uz}^S + \min_{\forall v_{ji}} \sum_{i=k}^{jg} W_{ji} - \sum_{i=k}^{jg} W_{ji}^{plan}. \quad (8)$$

In this formula jg is the index of the last line of train T_j route.

The formula (8) provides the optimization procedure for further movement of train T_j being overtaken. The procedure usually does not produce marked effect thus the formula (7b) may be applied instead of formula (8) with a slight error.

Situation C. To run line k general costs are:

$$^C W_{uk} = W_{uk}^E + N_u(\tau_{uk}) + W_{jk}^S. \qquad (9a)$$

To run the whole distance from station S_z to terminal station S_n of T_u route general costs are:

$$W_{uy} = \sum_{i=z}^{n}(W_{ui}) + W_{uz}^S + W_{jk}^S + \min_{\forall v_{ji}} \sum_{i=k}^{jg} W_{ji} - \sum_{i=k}^{jg} W_{ji}^{plan}.$$
$$(9b)$$

The formula (9b) also provides the optimization procedure for further movement of train T_j being overtaken. The procedure usually does not produce marked effect thus instead of the formula (8) the formula below may be applied with a slight error.

$$W_{uy} = \sum_{i=z}^{n}(W_{ui}) + W_{uz}^S + W_{jk}^S. \qquad (10)$$

W_{jk}^S involved in formulae (8) and (10) are the costs of arbitrary fuel necessary to develop the speed for train j after it was forced to stop.

In the case of terminal attraction W_{jk}^S is expressed as a formula:

$$W_{jk}^S = 0.515 \times 10^{-7}(v_{jk})^2 Q_j \left[102(1+\gamma) - \frac{w_{0j}}{a_T}\right], [\text{kg}]. \,(11)$$

Herein γ – inertia coefficient of turning mass, which depends on the type of train and loader and ranges form $\gamma = 0.028$ for loaded carriers to $\gamma = 0.084$ for empty carriers and $\gamma = (0.17–0.18)$ for locomotives;

a_T – Braking acceleration m/s² (in calculations $a_T = 0.22$ m/s² are constant value);

w_{0i} – expressed as formula (6).

D and E situations analytically are not complicated: additional conditions must be satisfied only:

$$v_{ui} \le v_{ji}, \quad i = k, k+1, ..., r, \qquad (12a)$$

(S_r – station of planned stopping for train T_j) in case **D** and other condition:

$$v_{ui} \le v_{ji}, \ i = k. \qquad (12b)$$

in case **E**

Situation F. To run the line k general costs are:

$$^F W_{uk} = W_{uk}^E + N_u(\tau_{uk}) + W_{uk}^S. \qquad (13a)$$

To run the whole distance from station S_z to terminal station S_n of T_u route general costs are:

$$W_{uy} = \sum_{i=z}^{n}(W_{ui}) + W_{uz}^S + W_{uk}^S. \qquad (13b)$$

Here W_{uk}^S is calculated according to the formula (11), but using characteristics of T_u train.

Formulae (7b), (8), (9b), (10) and (13b) assess costs involved in speed development of train T_u in primary line S_z only. Not always train T_u succeeds in interfering between trains' j and $(j+1)$. It is possible that seeking to keep a safe distance up to interfering train T_u, train $(j+1)$ will be forced to reduce the speed that results in delay for the arrival to the stations, starting from $(z+1)$. Due to this fact additional losses may be incurred (penalty) $W_{(j+1)}^P$, which are calculated as follows:

$$W^P_{(j+1)} = \sum_{z+1}^{(j+1)g} W^P_{(j+1)i}, \qquad (14a)$$

$$W^P_{(j+1)i} = 0, \text{ if } L_{j(j+1)i} > (L^{st}_{ui} + L^{st}_{(j+1)i}), \quad (14b)$$

$$W^P_{(j+1)i} = N_{(j+1)i}, \tau_{(j+1)i} = \frac{L_{j(j+1)i} - \left(L^{st}_{ui} + L^{st}_{(j+1)i}\right)}{v_{(j+1)i}},$$

$$\text{if} \quad L_{j(j+1)i} < (L^{st}_{ui} + L^{st}_{(j+1)i}). \qquad (14c)$$

Formulae (14) use these symbols:

L^{st}_{ui} and $L^{st}_{(j+1)i}$ – braking distance in the first line ($S_i - S_{(i+1)}$) respectively of train T_u and train ($j+1$), which is running after.

$L_{j(j+1)i}$ – distance between two neighbouring scheduled trains j and ($j+1$), where in line z train T_u is able to intervene in this distance;

$N_{(j+1)}(\tau_{(j+1)i})$ – losses, incurred as train ($j+1$) comes to station S_i at the delay time $\tau_{(j+1)i}$.

Technical speed of train ($j+1$) in line i is:

$$v_{(j+1)i} = \frac{l_i}{t^a_{(j+1)(i+1)} - t^i_{(j+1)i}}; \qquad (15)$$

l_i – length of the line.

If train ($j+1$) is not able to arrive later to the station S_i,

$$N_{(j+1)}(\tau_{(j+1)i}) \to \infty.$$

Additional losses (penalty) $W^P_{(j+1)}$ do not usually produce a marked effect thus in the algorithm below, concerned the calculating of general costs W_{uy}, they are not assessed.

Costs W_{uy} in accordance with the formulae (7)–(13) are calculated being aware of the parameters of traffic schedule and traffic participants:

1. Planned traffic schedule $\mathbf{S}(t)$ (see (3) formula).
2. Functions of delay losses (penalty) $N_j(\tau_{ji})$.
3. Train parameters G_j, M_j, N_j, Q_j, and parameters necessary to calculate w_{0ji} values.
4. Relief parameters $i_e(s)$.
5. Air temperature t^0.
6. Index z of the station, from which the line of train T_u is corrected (inserted).
7. Index p of the first train line, after which the correction line may be inserted.
8. Element values $c_{jk} \in \{1, \infty\}$ of "pass/ do not pass" matrix $\mathbf{C} = \|c_{jk}\|$, which may be given or pro-

duced accidentally for each iteration of optimization process.

9. Departure time t^i_{uz} of train T_u from the station S_z (chosen with regard to the application of regular or stochastic search algorithms, determinate order or at random) and technical speeds of all lines $v_u = \|v_{ui}\|$, (as $z \le i$ n-1).

By choosing the mentioned parameters, the restrictions should be satisfied as: $t^i_{(p+m)z} > t^i_{uz} > t^i_{pz}$

and $v^{max}_{ui} > v_{ui} > v^{min}_{ui}$.

The particular value of general costs W_{uy} is calculated for each fixed vector of variables.

It has to be noted that formulae (7)–(15) correspond to such interval system of train control, in which the distance between neighbouring trains may acquire any value (it is continuous). This is involved in the system of ETCS 3 level. If the distance between trains is expressed as the whole number of fixed blocked sectors, so the discrete variables of road, time and speed should be used in (7)–(15) formulae.

3. Algorithm for optimal correction of traffic schedule

The correction of the route is performed within permissible field (following the restrictions) seeking such vector of variable value $\{\ t^i_{uz}, \mathbf{v}_u, \mathbf{C}\ \}_{ger}$ when the value of general costs W_{uy} is minimal.

This problem may be solved by applying algorithms of stochastic search (Monte Carlo method, genetic search algorithm etc.), method of variations reselection (in respect of discrete variables) and other.

Monte Carlo algorithm is one of the simplest proposed hereinafter.

Begin

1. Parameters necessary to enter:

1.1. Traffic schedule $\mathbf{S}(t)$, functions of delay losses (penalties) $N_j(\tau_{ji})$, parameters of trains G_j, M_j, N_j, Q_j, w_{0ji}, relief parameters $i_e(s)$, air temperature t^0.

1.2. Index z of the station, from which train line T_u is corrected (inserted).

1.3. Index p of the first train line, after which the correction line may be inserted.

1.4. Optimization iteration number.

2. Restrictions necessary to enter:

2.1. $t^i_{(p+m)z} > t^i_{uz} > t^i_{pz}$.

2.2. $v^{max}_{ui} > v_{ui} > v^{min}_{ui}$.

3. Primary assigns:

3.1. $k := z$.

3.2. $j := p$.

3.3. $y := 1$.

3.4. $W_{ger} := \infty$.

4. Selection of random vector of variable value $\{\ t^i_{uz}, \boldsymbol{v}_u, \boldsymbol{C}\ \}$.

5. t^i_{uz} and $\boldsymbol{v}_u = \left\| v_{uji} \right\|$:

Yes $\rightarrow$ p. **6**,
No $\rightarrow$ p. **4**.

6. Subroutine "BRANDUOLYS"

7. If $W_{uy} < W_{ger}$

Yes $\rightarrow$ p. **8**,
No $\rightarrow$ p. **9**.

8. $W_{ger} := W_{uy}$.

9. Delete W_{uy} ($W_{uy} := 0$).

10. $y := y+1$.

11. If $y > N$

Yes $\rightarrow$ p. **12**,
No $\rightarrow$ p. **6**.

12. Enter W_{ger} and $\{\ t^i_{uz}, \boldsymbol{v}_u, \boldsymbol{C} \}_{ger}$.

End

Subroutine "BRANDUOLYS" realizes such algorithm:

Begin

1. $W_{uk} := 0$.

2. If $t^i_{(j+1)z} > t^i_{uz} > t^i_{jz}$

Yes $\rightarrow$ p.**4**,
No $\rightarrow$ p. **3**.

3. $j := j+1$, $\rightarrow$ p. **2**.

4. If $v_{uk} > v_{jk}$

Yes $\rightarrow$ p. **5**,
No $\rightarrow$ p. **19**.

5. Does T_u catch up Tj in line k

Yes $\rightarrow$ p. **13**,
No $\rightarrow$ p. **6**.

6. T$_j$ according to planned order stops at station S_{k+1}

Yes $\rightarrow$ p. **7**,
No $\rightarrow$ p. **8**.

7. T$_u$ overtake train T$_j$ standing at station S_{k+1}
Yes $\rightarrow$ p. **8**,
No $\rightarrow$ p. **9**.

8. W_{uk} calculated according to formula (7) $\rightarrow$ p. **18**.

9. T$_u$ disturbs train T$_j$ to leave station S_{k+1} timely.
Yes $\rightarrow$ p. **10**,
No $\rightarrow$ p. **11**.

10. W_{uk} calculated according to formula (8) $\rightarrow$ p. **12**.

11. W_{uk} calculated according to formula (7)$\rightarrow$ p. **12**.

12. $j := j-1$, $\rightarrow$ p. **18**.

13. Train T$_j$ which has been caught up by train T$_u$ will let it pass
Yes $\rightarrow$ p. **14**,
No $\rightarrow$ p. **15**.

14. W_{uk} calculated according to formula (9).$\rightarrow$ p. **12**.

15. T$_j$ according to planned order stops at station S_{k+1}
Yes $\rightarrow$ p. **16**,
No $\rightarrow$ p. **17**.

16. W_{uk} calculated according to formula (12b).

$\rightarrow$ p. **12**.

17. W_{uk} calculated according to formula (12a).
$\rightarrow$ p. **18**.

18. $W_{uy} := W_{uy} + W_{uk}$, $\rightarrow$ p. **23**.

19. If $v_{uk} < v_{(j+1)k}$

Yes $\rightarrow$ p. **20**,
No $\rightarrow$ p. **7**.

20. Does $T_{(j+1)}$ catch up Tu in line k

Yes $\rightarrow$ p. **21**,
No $\rightarrow$ p. **8**.

21. W_{uk} calculated according to formula (13).

$\rightarrow$ p. **22**.

22. $j := j+1$. $\rightarrow$ p. **18**.

23. If $k = n$
Yes $\rightarrow$ p. **25**,
No $\rightarrow$ p. **24**.

24. $W_{uk} := 0$. $\rightarrow$ p. **3**.

25. $W_{uy} := W_{uy} + W^S_{uz}$.

End

The 2nd, 4th and 19th conditions of the subroutine are checked with regard to the data of traffic schedule $\mathbf{S}(t)$ and the values selected for t^i_{uz} and

$\boldsymbol{v}_u = \left\| v_{uji} \right\|$ variables, the 13th condition is determined by the value of appropriate element of "pass/do not pass" $\boldsymbol{C} = \left\| c_{jk} \right\|$ matrix, which (subject to optimization algorithm) may be fixed or belong to the set of variables (in this case c_{jk} occasionally may acquire values "1" or "∞"). The checking of the remaining conditions of subroutine algorithm is concentred to the tasks of a line segment, being expressed as data of traffic schedule $\mathbf{S}(t)$ and selected values of t^i_{uz} and

$\boldsymbol{v}_u = \left\| v_{uji} \right\|$ variables, joint/disjoint tasks and search of the smallest distance between the segments of the line.

4. Conclusions

1. The tasks of train traffic correction and addition are solved having installed modern information control technologies.

2. It is reasonable to solve the problems of traffic schedule correction and optimal addition by applying the criterion of general costs. The solution is the optimal route line, which has minimal value of general costs.

3. The algorithms of traffic schedule correction and addition are subject to high "branch" level: there are a lot of conditions, which affect later actions and solutions.

4. The problems of traffic schedule correction and optimal addition are solved with the help of digital methods (by reselecting all combinations of discrete values of variable or by applying methods of stochastic search).

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