TRANSPORT
ISSN 1648-4142 print / ISSN 1648-3480 online
2012 Volume 27(4): 357–363
doi:10.3846/16484142.2012.750623

NUMERICAL RESEARCH OF THE RETAINING CONSTRUCTIONS DURING THE RECONSTRUCTION OF THE TRANSPORT STRUCTURES

Dmitry Prusov

National Aviation University, Kosmonavta Komarova ave 1, 03680 Kiev, Ukraine
E-mail: prusov@nau.edu.ua

Submitted 16 September 2011; accepted 25 November 2011

Abstract. The research analysis of interaction artificial retaining constructions with a non-uniform soil basis has been performed during the reconstruction of the transport structures. The mathematical model of soil semi-space research with the use of the nonlinear theory of elasticity is developed. The results of numerous researches of stress-strain state fencing constructions of the overpass in interaction with multilayer heterogeneous half-space and surfaces under the influence of band-pass loading are represented in the current paper.

Keywords: reconstruction, transport structures, stress-strain state, underground objects, analysis.

1. Introduction

Steady increase of population concentration in large cities and, as consequence, autopark growth generate the most urgent problem of modern cities – the territorial problem. World town planning practice shows that one of the most effective solutions to this problem is the development of the urban underground space. Foreign experience shows that for comfortable living in the metropolis fate of underground structures of the total area of objects that are introduced should be 20÷25%. Construction of underground facilities for various purposes has long been conducted around the world.

Underground construction of buildings and road transport network will organize a high-speed traffic and simultaneously increase the safety of pedestrians. Absence of delays in road transport traffic jams will reduce the cost of time around the town, reduce the level of traffic noise and air pollution levels.

Researches and calculations for the soil slopes design tend to perform by deformations. The sequence of geotechnical investigations, assessments, observations of deformations in nature should be subordinated to the optimal design framework, and design optimality criterion should act as the ratio of design and limit deformations. The condition for the first limit state should always be satisfied. On the other side, the condition for the second limit state is more flexible, so before the bedding soils begin to collapse, they have to observe deformation when the buildings or structures operating conditions are affected.

In the reconstruction process of the retaining constructions predicted value of additional strains of existing structures from the all kind impacts should be determined on the basis of comprehensive mathematical modeling by the finite element method using nonlinear models.

As analysis of theoretical and experimental investigations indicates, slip along soil-structure boundaries occurs for limiting values of tangential stresses, which do not attain the ultimate shear strength of the soil. Processes and mechanisms of contact interaction are modeled numerically, approximating the interface with finite elements of minimized thickness and utilizing a mathematical model reflecting the limiting shear and normal stresses on the boundary (Madzhidov 2001).

In the test configuration, a triangular failure wedge shape is observed, due to the low mobilized wall-soil friction (Wilson, Elgamal 2010). From the estimates, along with triaxial and direct shear test data, theoretical predictions are compared with the measured passive resistance. Using the test data, a calibrated finite-element model is employed to produce additional load-displacement curves for a wider range of practical applications.

The plane strain finite element analysis has been carried for studying the behavior of a piled bridge abutment on a soft ground (Wang *et al.* 2007). In this model, the consolidation of soil is realized by employing couple pore fluid diffusion and stress procedure which models soil as saturated porous media and uses pore pressure element at the same time. For better conforming the intrinsic 3D nature of the structure and soil-structure interaction, some measures are taken when constructing the plane strain model, and the success of this model has been verified by data from onsite experiments.

The FEM stability analysis has been applied to simulate the stress and strain relationship of the soil slope with different constitutive nonlinear anisotropic models, and the displacement criterion has been used to compute the factor of safety (Zhang *et al.* 2011).

The Road Retaining Wall as an important part of the transportation system has been considered for post-earthquake emergency safety assessment (Li *et al.* 2010). Safety criteria are established and correlations of such criteria with various damage states are given. This paper presents the methods followed by the evaluation process, highlights the items for evaluation and intends to provide technical references for the post-earthquake disease prevention, diagnosis and disease treatment.

The basic series of models has been designed to use in calculations for schemes of finite element method, which allows the modeling to solve problems within the limit values considering the real behavior of soils. In implementing this scheme the patterns of relationships that account the complex soil behavior should be consistent with two-dimensional and three-dimensional finite elements, and also describe the behavior soil reaction and consider models for different types of influences. These problems of determining some parameters are rather difficult and intended for complex special theoretical and practical researches outside the engineering calculations.

Analysis of recent researches and publications indicates that the existing rules and empirical dependences on evaluating the stability of soil mass lead to unnecessary safety margin or, on the contrary, to their destruction, and some modern construction technology is new and not yet reflected in the regulations and standards.

Modern approaches to the analysis of these systems assume it's only means of numerical modeling that requires the use of modern computer base and appropriate mathematical apparatus. The complexity of solving the relevant problems consists not only in the creation or use of appropriate techniques, but primarily in making reasonable physical model that most correctly describes the nonlinear deformation processes of the material environment, including soil material model, as well as in the choice of the design schemes and implementation of the special calculation algorithms that ensure the accuracy of calculation results. The situation is complicated by the fact that there is no universal method or model that can be applied to any geological environment.

Thus, each problem statement should include its own analysis of authenticity and a special approach, which is the urgent problem of structural mechanics, modern design and numerical modeling to provide a reliable and economic solution of a problem.

The scientific novelty of this work is in the development of the patterns relationships that are derived from generalized dependencies of structural mechanics and soil mechanics, that allow to determine more reasonably the stress-strain state of retaining fencing structures and adjacent bedding foundations, depending on soil heterogeneities. This determining approach of the soil design characteristics differs from previously known that it allows to consider the heterogeneity of natural soil bedding, physical and mechanical characteristics of discrete soil elements.

2. Problem Statement

Rapidly increasing traffic flow leads to the necessity of the road network development, so for large mega cities building of underground transport constructions is especially actual problem.

The last researches were of interaction of the road pavements soil basis with artificial fencing constructions connected with definition of the stress-strain condition and firmness of a soil massive which is propped up by these structures. When designing the road structures on sites with difficult soil conditions, for prevention of the soil soaking the device of a waterproofing layer under road pavement is supposed. But these actions in practice often happen ineffective. For the solution to this problem, the design procedure of elastic semispace that is used in research of interaction of a soil basis with artificial retaining constructions is considered (Prusov 2007; Prusov, Minakova 2010; Prusov, Beljatynskij 2011).

The specified methodology is based on the finite elements method (FEM) that is one of the most effective numerical network methods. In setting, the problem modeling has been supposed to be the heterogeneous of soil semispace layers.

Estimation of stress state semispace has been involved in the comparison of calculation results of extremely allowable values of deformations and displacements, i.e. the loss of stability in the local areas and the development of plastic deformations are possible. Due to these facts the plane problem formulation of a nonlinear deformable solid has been considered as the geometric and physical nonlinearity, and has been used as the moment scheme of finite elements (MSFE) (Bazhenov *et al.* 2000, 2002).

The initial value MSSE based on continuum mechanics has been considered using the relationships of analytical nonlinear theory of elasticity and plasticity in increments. The application of elasticity theory for solving soil mechanics problems has provide an accurate description of the stress-strain state of soil space for critical and post-critical conditions, considering the characteristics that reflect real physical and mechanical properties of soils.

3. Mathematical Model of a Problem

The finite elements method is among the energy methods, therefore for a problem statement as a theoretical basis is used the theory of nonlinear mechanics of soils on the base of the continuum mechanics with application of relationships in the increments of movements, deformations and stresses. Subject to the nonlinear elasticity theory relationships first principle of virtual work for static problems in the actual configuration of three-dimensional nonlinear deformable body can be written as follows:

$$\int_v \left(\sigma'^{ij} + C^{ijkl}_{(e,p)} \gamma_{kl} \right) \delta\gamma_{ij} dv - \int_v p^{i'} \delta u_{i'} dv - \int_S q^{i'} \delta u_{i'} dS = 0, \quad (1)$$

where: σ'^{ij} – is a component of initial stress tensor; $C^{ijkl}_{(e,p)}$ – is a component of elastic tensor of a solid considering the development of elastic-plastic deformations; γ_{ij} – variations of covariant increments components of finite deformations tensor in local system; $p^{i'}$, $q^{i'}$ – components of the generalised vectors of volume and surface forces in global coordinate system $OZ^{1'}Z^{2'}Z^{3'}$, that act on a body in an actual configuration and carried to reference configuration; $\delta u_{i'}$ – variations of a vector components of increment displacements in global Cartesian coordinate system.

Variation equation (1) according to energy methods describes equilibrium of elementary volume of any whole continuum, independently of its physical properties. This representation realizes a practical conclusion of variation problems of the elasticity theory and the limits of stress state theory, where solutions are connected with isolation of moved plastic deformations zones (e.g., for soils) (Harr 1971). Thus, the determined problem of the statics of soil semispace is considered and in each point (nodes of discrete model and borders between them) conditions of a limiting intense condition are satisfied (Tsytovich, Ter-Martirosyan 1981).

The variation equation (1) is valid for the decision of plane problems of the nonlinear theory of elasticity (a plane stress state and plane deformation).

The heterogeneous semispace is considered, which is modeled by a discrete reflection as a set of finite elements, each of that is homogeneous solids with different physical and mechanical characteristics of isotropic or orthotropic bodies, but on the boundaries of finite elements the condition of deformations compatibility is satisfied, i.e. these comes compatible FE ensuring uneven grid area for approximation with some degree of accuracy the stress concentration zones and the development of shear deformation due to the using of finite strain tensor, that describes the displacements and rotations of FE as a rigid unit (Shimanovsky, Tsyhanovsky 2005).

Natural limiting conditions (rigid) are used, that special limiting conditions are realised by imposing of rigid bracing on boundaries of discrete design model, and also special boundary conditions with introduction of the equations of geometrical kinematical conditions with realization of a variation problem solutions with a method of uncertain Lagrange's multipliers when the variation equation of discrete model (1) is supplemented with the equation of deformation conditions of this model through system of geometrical constraints.

In the given work the criterion of stability or plasticity of soil semispace for separate local homogeneous isotropic area, is offered in the most universal form on the basis of expanded Mises' criterion (at the expense of inclusion at it the dependence on hydrostatic pressure), with the use of a surface loading through the Mohr-Coulomb model, and considering the third invariant of stress tensor-deviator function through Lode-Nadai invariant (Tsyhanovs'ky, Prusov 2004). For the reason that stress tensor invariants are defined through components of spherical part and deviator part of stress function and an assumption about uniformity and isotropic local semispace locality makes it independent of the direction of octahedral planes normals and the expanded modified Mises' fluidity criterion can be resulted in the following form:

$$f\left(\hat{\sigma}, \hat{\gamma}^{(p)}, \alpha, \varphi, c\right) = \frac{2}{3} I_1\left(\hat{S}^2\right) \times \left(\cos\alpha - \frac{1}{\sqrt{3}} \sin\alpha \sin\varphi\right)^2 - \left(\frac{1}{\sqrt{3}} I_1\left(\hat{\sigma}\right) \sin\varphi - \sqrt{3}\, c \cos\varphi\right)^2 = 0, \quad (2)$$

where: $\hat{\sigma}$, $\hat{S}$, $\hat{\gamma}^{(p)}$ – are the tensors accordingly to general stresses, stresses of deviator part and plastic deformations; α, φ, c – accordingly the stresses invariant – the Lode-Naday's angle, the angle of soil internal friction, and the soil specific cohesion.

Using the plastic flow theory within the limits of expanded Mises' principle, on the basis of associated law it is possible to write down:

$$C^{ijkl}_{(e,p)} = C^{ijkl}_{(e)} - \beta n^{ij} n^{kl};$$
$$n^{ij} = C^{ijkl}_{(e)} f_{,\hat{s}kl}; \quad (3)$$
$$\beta = \left(n^{ij} f_{,\hat{s}ij}\right)^{-1}; \ \sigma^{ij} = C^{ijk}_{(e,p)} \gamma_{kl};$$
$$\gamma_{kl} = \gamma^{(e)}_{kl} + \gamma^{(p)}_{kl}. \quad (4)$$

For reception of relationships (3), (4) it is necessary to differentiate the scalar function (2) on tensor argument $\hat{S}$, as a complex function.

The discrete model of soil semispace of an individual thickness is presented by a set of two-dimensional quadrangular curvilinear finite elements (in general), in each of which is entered local oblique-angled basis $\vec{e}_i$ with the beginning in the centre FE. There are accepted nodal displacements of FE in global coordinate system $OZ^{1'}Z^{2'}Z^{3'}$. Changes of displacements in limits of FE are defined by polylinear functions of two coordinates of a soil semispace surface X^1, X^2:

$$u^{i'} = \sum_{s_1=\pm1} \sum_{s_2=\pm1} u^{i'}_{s_1 s_2} \prod_{\delta=1}^{2} \left(S_{(\delta)} x^{(\delta)} + \frac{1}{2} \right), \quad (5)$$

where: $S_\delta = \pm1$ – conditional Lagrange's coordinates; $u^i_{S_1 S_2}$ – central displacements; $-1/2 \le x^\delta \le 1/2$.

Approximation of displacements in FE limits and form function coincide, so, a finite element which is used, is isoparameter.

The system of nonlinear equations of FE model equilibrium of plane semispace is derived from expression of a variation of full potential energy for the one finite element:

$$\delta\Pi_E = \sum_{P_1=\pm1} \sum_{P_2=\pm1} \left(R^{t'}_{P_1 P_2} - Q^{t'}_{P_1 P_2} \right) \delta_{P_1 P_2} u_{t'}. \quad (6)$$

System of the nonlinear equations of FE semispace model equilibrium is true for the solving of plane problems of nonlinear elasticity theory. Problems solutions to plane stress state and plane strain depend on the physical and mechanical characteristics of the material used. For an isotropic solid in case of a plane stress state it is:

$$g_{\alpha 3} = g^{\alpha 3} = G_{\alpha 3} = G^{\alpha 3} = 0; \quad \gamma_{\alpha 3} = 0; \quad \gamma_{33} \neq 0; \tag{7}$$

$$\sigma^{33} = 0; \tag{8}$$

$$\sigma^{\alpha\beta} = B^{\alpha\beta\chi\varepsilon}\gamma_{\chi\varepsilon}; \tag{9}$$

$$B^{\alpha\beta\chi\varepsilon} = \frac{E}{1+\nu}\left(\frac{\nu}{1-\nu}G^{\alpha\beta}G^{\chi\varepsilon} + \frac{1}{2}\left(G^{\alpha\beta}G^{\chi\varepsilon} + G^{\alpha\varepsilon}G^{\chi\beta}\right)\right). \tag{10}$$

In case of plane strain it is:

$$g_{\alpha 3} = g^{\alpha 3} = G_{\alpha 3} = G^{\alpha 3} = 0; \quad \gamma_{\alpha 3} = 0; \quad \gamma_{33} = 0; \tag{11}$$

$$\sigma^{33} \neq 0; \quad \sigma^{33} = \nu\left(\sigma^{11} + \sigma^{22}\right). \tag{12}$$

To prove the reliability of this methodology for the elastic semispace investigation a number of tests and control tasks are solved (Tsyhanovs'ky, Prusov 2004).

The soil semispace research methodology is used for the nonlinear elasticity theory which provides reliable results of the solutions to soil mechanics plane problem considering heterogeneity of the semispace, the presence of layers with different physical and mechanical characteristics, different boundary conditions, random external influences and also makes it possible to calculate soil semispace with the inclusion of structural elements of foundations, retaining walls and other strengthening structures.

4. Numerical Researches

This methodology is used for research of the stress-strain state of retaining walls with nonhomogeneous soil semispace in the reconstruction design of the auto-road overpass in Donetsk, Ukraine (Fig. 1).

The plane research problem of the stress-strain state of strengthening fencing overpass structures is considered in interaction with multilayered non-uniform semispace and pavements with the influence of a strip loading which models the vertical wheel loading from a vehicle.

Taking into account the symmetry of loading in a cross-section direction of the overpass constructions, as a design scheme is accepted, the half width of the overpass, i.e. 9 m width and 12 m length of the road carriageway, height it is accepted 6.5 m (overpass height), 6.5 m (height of an underground part of the piles) and 3.5 m (soil space) (Fig. 2).

Fig. 1. The general view of the auto-road overpass and the strengthening fencing structures

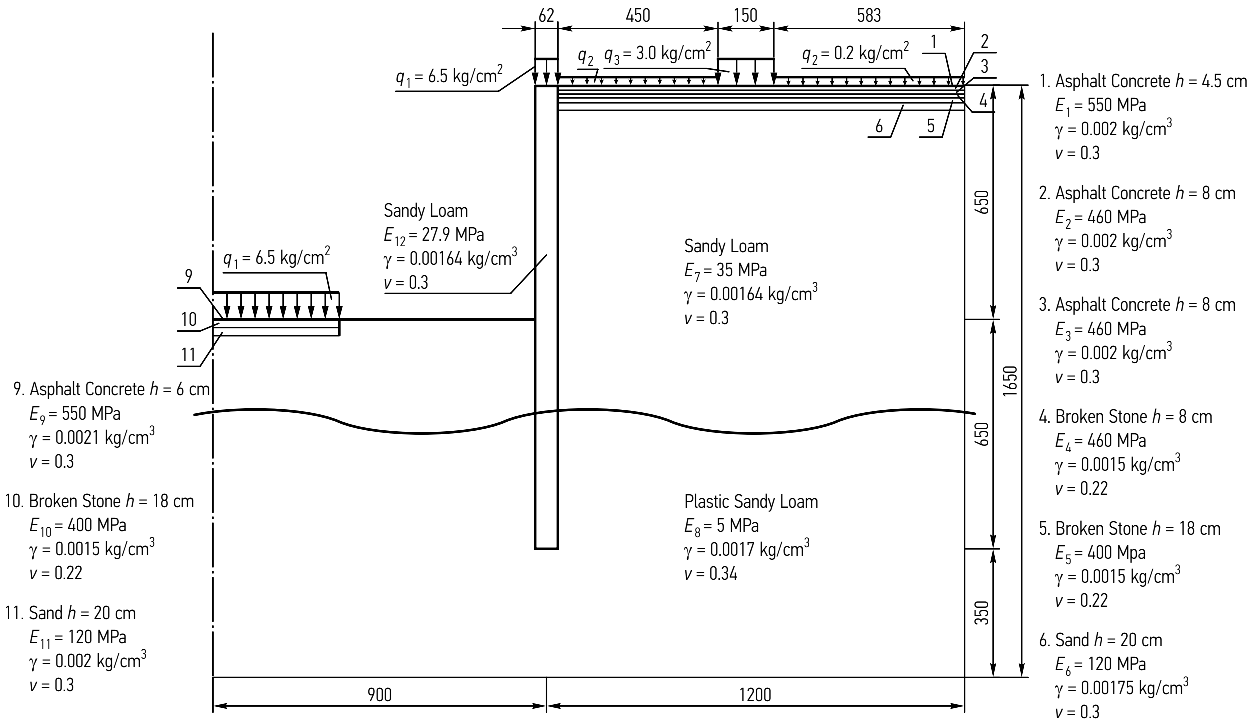

Fig. 2. The constructive scheme of the overpass retaining structures with the soil basis interaction

External normative and calculated loadings of pavement weight, constructions self-weight and snow are represented $Q = 6.5$ kg/cm^2. On distance 5.12 m from the edge of fencing constructions the loading is applied from a vehicle that is equal to 3.0 kg/cm^2. Modeling of equivalent strip loading is reduced to the pressure distribution intensity that is equal to $Q = 0.2$ kg/cm. The unfavourable situation is in the presence of applied loadings on the main roadway and the absence of applied loadings under the overpass.

Net area of the design scheme has regular mixed steps for the description providing of the overpass structures with fencing strengthening constructions and soil bedding layers by the FE-modeling.

Taking into account the thickness value of compressing semispace strata, the discrete model has the sizes 165×210 cm (h).

Based on the used finite element moment scheme the net area with dimensions M2×M3 – 29×29 need to be chosen, that is corresponding the discrete model which includes 841 FE, considering the cavity which models the soil excavation under the overpass. The problem solution algorithm has provided the absence of cavity at the first stage, lengthening on the network coordinates parametres S_3 which are carried out at the second stage, the cavity is generated consistently, beginning from the top plane of semispace within network coordinates S_2 =17÷29 for modeling of excavation evolution under the overpass (Fig. 3).

The overpass with the area 697.5 m^2 and depth 6.5 m has a fencing construction type 'the wall in the ground' with thickness value 0.62 m. Wall height in the ground – 13 m. According to conditions of the real object building the soil semispace consists of layers with the different physical and mechanical characteristics: non-rigid asphalt concrete pavement (3 layaers) 61 cm, E = 550 MPa, ν = 0.3, γ = 0.0021 kg/cm^3; broken rock granite saturated with bitumen (R_{cr} = 80MPa) – 8 cm, $E = 7{\cdot}10^2$ MPa, ν = 0.3, γ = 0.0020kg/cm^3; broken stone – rock granite – 18 cm, $E = 3.5{\cdot}10^2$ MPa, ν = 0.22, γ = 0.0015 kg/cm^3; sand (last layer of artificial base of non-rigid pavement) – 20 cm, E = 120 MPa, ν = 0.3, γ = 0.00175 kg/cm^3; sandy loam (beginning of soil base) – E = 35 MPa, ν = 0.3, γ = 0.00164 kg/cm^3; plastic sandy loam – E = 35 MPa, ν = 0.34, γ = 0.00171 kg/cm^3.

Results of the numerical calculations have been received on the basis of discrete finite-element model, and show the character of plastic deformations development which is indicated in the Fig. 3 by '×' symbol. These plastic deformations concentrate along the wall in a ground with further increase in directions of strip loadings and soil vent (Fig. 3).

Using the methodology for this problem the results of numerical calculations have been obtained for the two phases that are associated with the technological characteristics of excavation of the real problem: phase I – the soil excavation up to the depth of – 3.50 m; phase II – the soil excavation up to the depth of – 6.50 m.

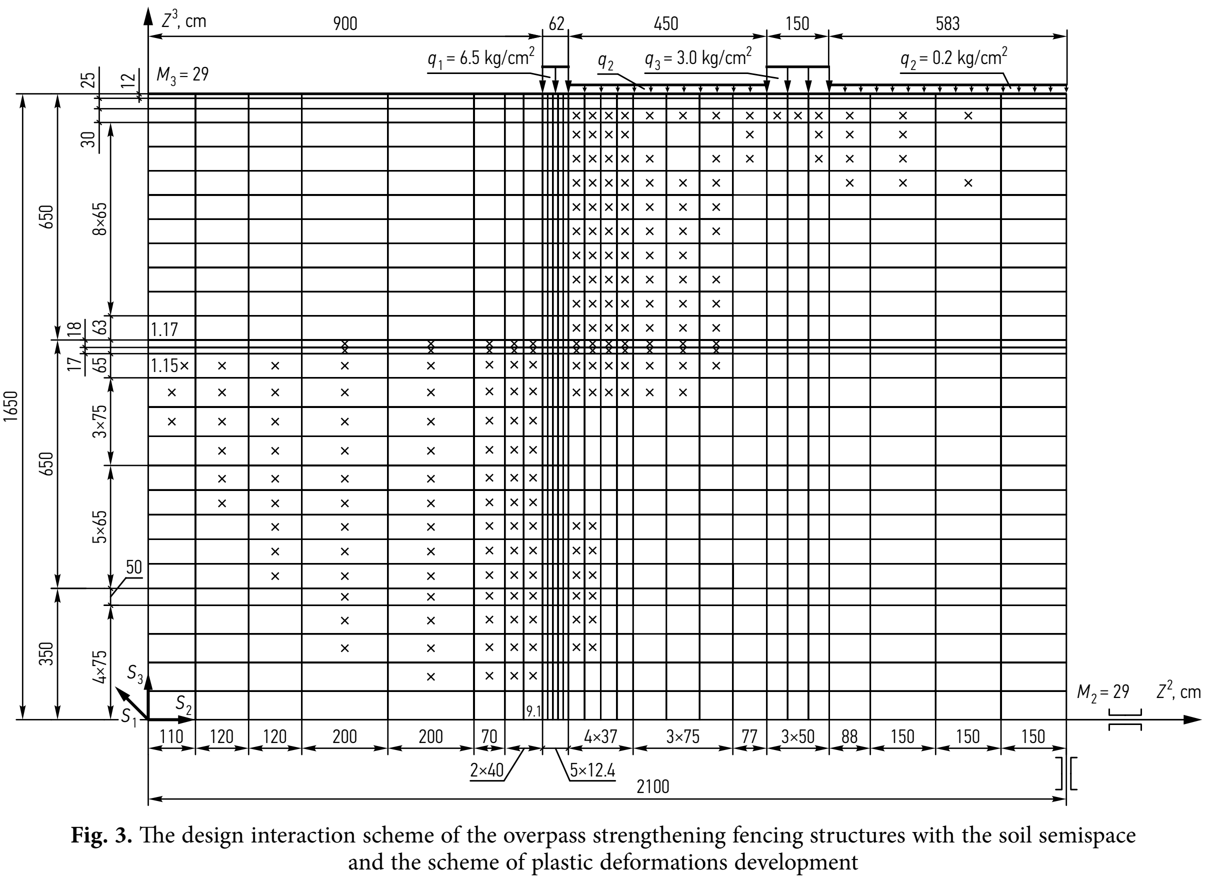

Fig. 3. The design interaction scheme of the overpass strengthening fencing structures with the soil semispace and the scheme of plastic deformations development

The results of the numerical calculations by the displacements of the retaining structure in the second direction of the global coordinate system $OZ^1Z^2Z^3–U^2{}_N$, and also by the bending moments of the retaining wall have been presented in Fig. 4 as the displacements characteristic diagrams of the retaining structure and the excavation bottom. The character of internal efforts diagram corresponds to the actual construction work.

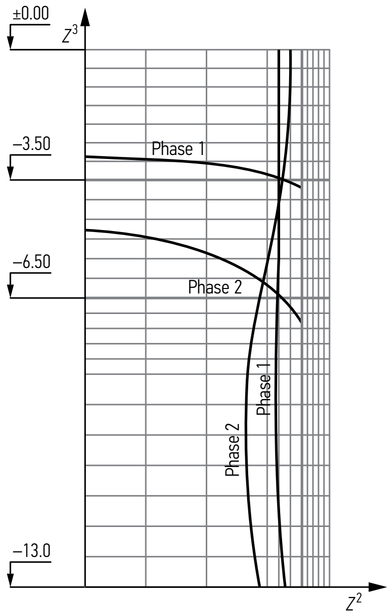

Fig. 4. The displacements characteristic diagrams of the retaining structure and the excavation bottom

The results of the numerical calculations by the displacements in characteristic points of the retaining structure and the excavation bottom, and also by the relevant bending moments have been compared with the results of the engineering calculations, considering the two phases of the soil excavation (Table).

Table. The values of horizontal displacements U_N^2 (mm) and the values of bending moments M_N^{33} (kNm/m) in characteristic points of the retaining structure

		Engineering Calculations	The Methodology
Phase I	U_N^2	1.20	1.46
	M_N^{33}	39.50	45.60
Phase II	U_N^2	3.70	4.63
	M_N^{33}	112.60	135.55

Comparison of the numerical researches results with engineering calculations shows the difference between the results obtained using this methodology (18% and 20% for displacements, 13% and 17% for bending moments respectively in the phase I and phase II) due to more accurate accounting the stress-strain state of the soil semispace in interaction with the overpass strengthening fencing structures.

The analysis of the stress-strain state of the soil massive and the overpass retaining piles constructions show that internal efforts in construction elements and its deformations do not exceed the permissible values, and allow to keep the overpass pavement from destruction.

On the basis of the received results of the spent researches of interaction of a soil basis with the artificial fencing constructions specified optimum sizes has been clarified for prevention of the loss of a soil stability, and also developments of soil vent and soil suffusion processes at the reconstruction and the further operation of the overpass.

5. Conclusions

The research methodology of soil semispace with using the nonlinear elasticity theory provides reliable results for a plane problem solutions of soil mechanics considering the semispace heterogeneity, presence of layers with the different physical and mechanical properties, different boundary conditions and any external loadings and influences.

Theoretically grounded and confirmed, the research possibility of the flat inhomogeneous soil semi-space is based on the nonlinear theory of elasticity and plasticity using the incremental theory and developing the theory of limit stress state of soil, considering geometrical and physical nonlinearity in the strains and stresses increments with the introduction of the extended Mises' fluidity criterion.

It is possible to carry out calculations for interaction research of a soil basis and constructive elements of various profound constructions in difficult engineering-geological conditions on the basis of the presented methodology for research, analysis, and design procedures of soil semispace with inclusions of constructive elements of the foundation bases, retaining walls and other strengthening fencing constructions.

The results of this research have been adopted for the use in calculating the retaining walls when designing of the real object during reconstruction of the auto-road overpass in Donetsk, Ukraine. The purpose has been to clarify the behavior of filler constructions in interaction with heterogeneous soil half-space, and to analyze the stress-strain state of fencing constructions of this object considering the real difficult complex engineering geological conditions and identify potential hazards. Implementation results of this research has been allowed in some measure to reduce the possibility of soil mass displacement and the risk of destruction of structures strengthening, and to improve the reliability of further exploitation of the object after reconstruction.

References

Bazhenov, V. A.; Saharov, A. S.; Tsyhanovsky, V. K. 2002. Momentnaia shema metoda konechnykh elementov v zadachakh nelinejnoj mekhaniki sploshnoj sredy, *Prikladnaya Mekhanika* 38(6): 24–63 (in Russian).

Bazhenov, V. A.; Tsyhanovs'ky, V. K.; Kislooky, V. M. 2000. *Metod skinchennych elementiv u zadachah nelinijnogo deformuvannia tonkih ta m'jakyh obolonok*. Kyiv, KNUBA. 386 s. (in Ukrainian).

Harr, M. E. 1971. *Osnovy teoreticheskoj mechaniki gruntov*. Moskva: Strojizdat. 320 s. (in Russian).

Li, J.; Li, Z.; Zhang, H. 2010. Post-earthquake emergency safety assessment of road retaining wall, in *ICTIS 2011: Multimodal Approach to Sustained Transportation System Development: Information, Technology, Implementation: Proceedings of the First International Conference on Transportation Information and Safety,* held in Wuhan, China, June 30 – July 2, 2011, 174–181.
http://dx.doi.org/10.1061/41177(415)22

Madzhidov, I. U. 2001. Numerical modeling of the interaction between a soil mass and concrete structures, *Hydrotechnical Construction* 35(5): 225–226.
http://dx.doi.org/10.1023/A:1012369809821

Prusov, D. E. 2007. Modeliuvannya napruzheno-deformovanogo stanu neodnoridnogo gruntovogo masyvu u vzaemodiji z zalizobetonnymy konstrukciyamy ukriplen', *Zbirnik Naukovyh Prac' Lugans'kogo Nacional'nogo Agrarnogo Universitetu* 71: 34–43 (in Ukrainian).

Prusov, D.; Beljatynskij, A. 2011. Zavisimost' sostoianiia zaglubliоnnykh konstrukcij ob'jektov aeroportov ot granichnogo ravnovesiia gruntovogo poluprostranstva, *Mokslas – Lietuvos ateitis* [Science – Future of Lithuania] 3(2): 118–125. http://dx.doi.org/10.3846/mla.2011.042 (in Russian).

Prusov, D. E.; Minakova, A. O. 2010. Porivnyal'nyj analiz stijkosti gruntovyh ukosiv za rozvytkom oblastej lokal'nyh zsuviv v jih osnovi, *Visti Avtomobil'no-Dorozhniogo Institutu* 2: 116–124 (in Ukrainian).

Shimanovsky, A. V.; Tsyhanovsky, V. K. 2005. *Teoriia i raschiot sil'nolinejnykh konstrukcij*. Kyiv. Stal'. 432 s. (in Russian).

Tsyhanovs'ky, V. K.; Prusov, D. E. 2004. Metod skinchennyh elementiv u zadachah doslidzhennya neognoridnogo pivprostoru z urahuvannyam geometrichnoji i fizichnoji nelinijnosti, *Opir Materialiv ta Teoriya Sporud* 75: 87–98. (in Ukrainian).

Tsytovich, N. A.; Ter-Martirosyan, Z. G. 1981. *Osnovy prikladnoj geomekhaniki v stroitel'stve*. Moskva: Vysshaia shkola. 317 s. (in Russian).

Wang, H. T.; Chen, Z. P.; Xiao, L. J. 2007. Plane strain finite element analysis of a piled bridge abutment on soft ground, in *Computational Methods in Engineering and Science: Proceedings of Enhancement and Promotion of Computational Methods in Engineering and Science X,* August 21–23, 2006, Sanya, China, 235–235.
http://dx.doi.org/10.1007/978-3-540-48260-4_81

Wilson, P.; Elgamal, A. 2010. Large-scale passive earth pressure load-displacement tests and numerical simulation, *Journal of Geotechnical and Geoenvironmental Engineering* 136(12): 1634–1643.
http://dx.doi.org/10.1061/(ASCE)GT.1943-5606.0000386

Zhang, K.; Li, W.; Shi, J. 2011. FEM stability analysis on soil slope with different constitutive models, in *Slope Stability and Earth Retaining Walls: Proceedings of the Geohunan International Conference II: Emerging Technologies for Design, Construction, Rehabilitation, and Inspection of Transportation Infrastructure*, June 9–11, 2011, Hunan, China, 26–33.
http://dx.doi.org/10.1061/47627(406)4