



RANDOM COEFFICIENT MODELING RESEARCH ON SHORT-TERM FORECAST OF PASSENGER FLOW INTO AN URBAN RAIL TRANSIT STATION

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Abstract. Taking a representative metro station in Beijing as example, this research has newly developed a random coefficient model to predict the short-term passenger flows with sudden increases sometimes into an urban rail transit station. The hierarchical Bayesian approach is iteratively applied in this work to estimate the new model and the estimation outcomes in each of the iterative calibrations are improved by sequential Bayesian updating. It has been proved that the estimation procedure is able to effectively converge to rational results with satisfying accuracies. In addition, the model application study reveals that besides sufficient preparations in manpower, devices, etc.; the information of the factors affecting the passenger flows into an urban rail transit station should be timely transferred in advance from important buildings, road intersections, squares and so on in neighborhood to this station. In this way, this station is able to cope with the unexpectedly sharp increases of the passenger flows into the station to ensure its operation safety.

Keywords: random coefficient modeling; Bayesian estimation and updating; short-term forecast; passenger flow; urban rail transit station; operation safety.

Introduction

The amount of the passengers entering an urban rail transit station is very likely to have a substantial increase in almost only a few minutes as a result of a large-scale entertainment or commercial activity, a sudden change in weather and so on (Tian *et al.* 2013). Such a passenger volume outburst is easy to cause operation safety accidents of a rail transit station if its response measures are not sufficient or timely enough due to a poor early warning. Therefore, a successful short-term forecast of the passenger flows into a rail transit station is crucial in particular for a busy urban rail transit station to take countermeasures in advance to adequately improve its safety capacity in time in case of an unexpected travel demand increase.

Short-term forecasting techniques in transportation field have received a widespread attention from researchers and engineers (Ma *et al.* 2014). Various methods including Wavelet Analysis (Huang 2003), Time Series (Williams, Hoel 2003), Bayesian Network (Sun *et al.* 2006), Fuzzy Neural Networks (Tsai *et al.* 2009), Support Vector Machine (Zhang, Liu 2009) and so forth have been applied for rational predictions of

short-term changes of travel demands. Different forecasting approaches can be generally categorized into parametric and non-parametric techniques referring to the functional dependency assumed between independent and dependent variables (Vlahogianni *et al.* 2004; Wei, Chen 2012). Until now, it is still controversial to say which approach is able to globally obtain best predictive performance among alternatives (Tsai *et al.* 2009). However, it has been recognized that short-term shocks on the prediction trend component frequently cause deviations of demand from its underlying growth path (Scarpel 2014). Accordingly, various efforts have been made to avoid such deviations. For instance, Ozyildirim *et al.* (2010) employ leading indicators to anticipate changes on future travel demand trend with cyclical characteristics. Moreover, Chen and Wei (2011) make use of the Hilbert–Huang transform (Huang, Attoh-Okine 2005) to analyze time variants of short-term passenger flow in a metro system. In a further, hybrid models are commonly used today for the improvement of the short-term forecast accuracy (Zhang *et al.* 2006; Tan *et al.* 2009; Zhang, Liu 2011; Wei, Chen 2012; Ma *et al.* 2014; Xie *et al.* 2014).



Although many relatively effective short-term forecasting approaches have been put forward, the predictions without the assumptions of linearity and stationarity on the short-term passenger volume changes still need further explorations especially for an urban rail transit station with huge passenger flow pressures. In other words, owing to its random characteristics, a sudden change of a short-term travel demand (such as a passenger flow into an urban rail station) is very hard to be rationally predicted. As a result, this research develops a random coefficient model estimated by hierarchical Bayesian approach to forecast the short-term passenger flows with abrupt increases into an urban transit station from the nonlinearly dynamic perspective of random changes. As a busy metro station which has usually huge passenger flows with frequent unexpected short-term increases especially on weekends, Xidan Station (XS) on Beijing metro network has been taken as example in this work. In view of their dynamically sharp increases in comparison to the relative stability of the passenger flows mainly consisting of the commuters into XS during weekdays, the passenger flows into XS on Sunday have been studied in detail.

The contents of this paper are organized as follows. Section 1 introduces XS and the modeling data surveyed in the area of XS. Section 2 makes the random coefficient modeling research on the short-term forecast of the passenger flows into XS. According to the forecast model developed in Section 2, the extreme cases about the sharp increases of the passenger flows into XS are analyzed in Section 3. Finally, last section makes conclusions and indicates some future research issues.

1. Data Survey

The urban rail transit lines in Beijing have got a total length of 442 kilometers till the end of 2012 and 261 stations have been put into operation (Guo *et al.* 2013). XS located in a very prosperous business district of Beijing is the transfer station of Line No. 1 and Line No. 4 on Beijing metro network. This metro station is surrounded by department stores, entertainment venues, banks, office buildings, and so on. Owing to the big sales in the department stores usually on weekends and holidays, the flows of the metro passengers arriving at and departing from XS in the afternoon of such a day always become huge. Meanwhile, especially the volume of the passengers entering XS is very likely to have a relatively sharp increase in a short time interval. Due to such quick increases of big passenger flows into XS from the buildings in neighborhood, sometimes XS had to strictly limit the number of the passengers entering the station to reduce the risks of its operational security if the increases are sustained. On account of the big passenger flows into XS and their often sudden increases, the volumes of the passengers entering XS through its main entrances in various time periods on Sunday have been surveyed for the modeling study in this research.

The survey was made on 27 October 2013. According to the introduction of the management staff of XS in the survey, the entrance F1 and entrance B of XS representatively face up to the intense passenger flows into XS

on weekends and holidays. Furthermore, the passenger flows into XS before 11:30 and after 20:00 in such a day are extremely small. Therefore, the volumes of the passengers entering XS through these two entrances have been recorded in successive short time intervals (i.e., 5 minutes in this study) which are smaller than the smallest time span of the successive sudden increases of the passenger flows from 11:30 to 20:00. Moreover, making important contributions to the passenger flows into XS, the related factors (i.e. the amounts of the people utilizing the escalators rolling downstairs from the second floors in two neighboring department stores named Juntai and Hanguang respectively, and the people walking towards XS from a nearby road intersection represented by RI in this study) have also been investigated in every 5 minutes from 11:30 to 20:00 for the modeling study in this work.

It is presented in Fig. 1 that the volumes of the passengers entering XS through F1 and B both generally first increase and then decrease with time passed. In the whole process, suddenly considerable increases of the passenger volumes obviously happen from time to time. The value changes of the related factors in every 5 minutes, as indicated in Fig. 2, follow similar trends of those of the passenger volumes shown in Fig. 1. It is also found that the sudden value increases of each of the related factors always take place at some time correspondingly before those of either the passenger flows. The relative locations of F1, B, Juntai, Hanguang and RI and the passenger flows into F1 and B from Juntai, Hanguang and RI are approximately described in Fig. 3. It is revealed that Juntai and Hanguang principally affect the volume of the passengers entering F1 and the passenger flow into B is primarily affected by RI. According to the field survey, the time used for walking to B and F1 from RI and the escalators rolling downwards to the first floor in Juntai and Hanguang respectively is basically equal to the time between the sudden increases of the passenger flows and the corresponding increases of their related factors' values.

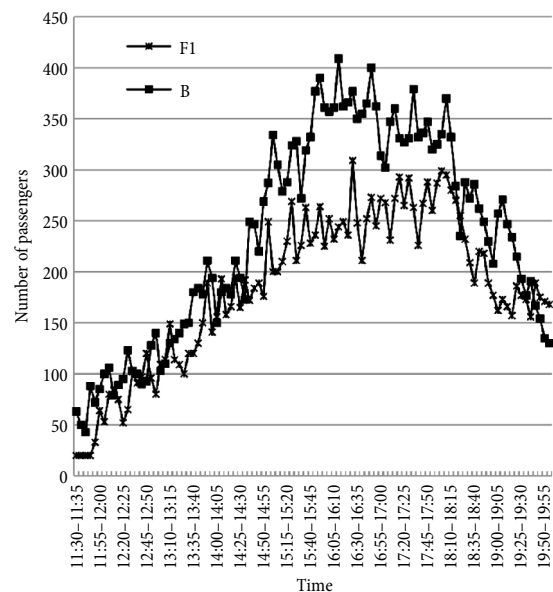


Fig. 1. Changes of the passenger flows into XS with time

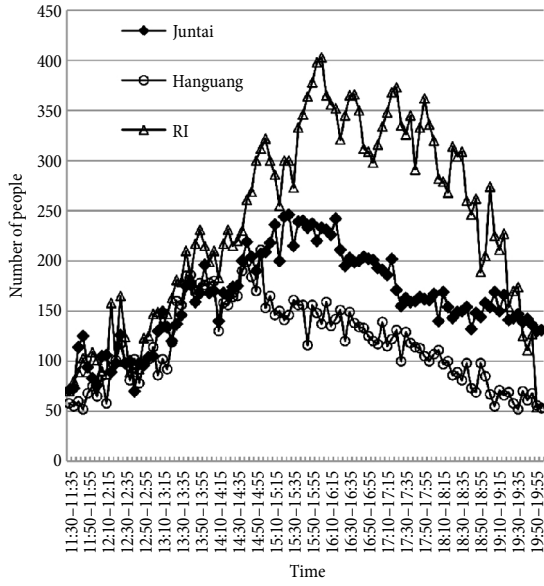


Fig. 2. Changes of the values of the related factors with time

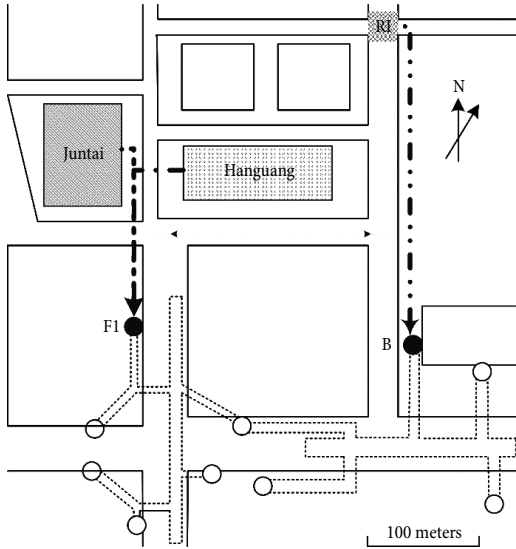


Fig. 3. Locations of F1, B, Juntai, Hanguang and RI and the passenger flows

2. Modeling Study

It is interpreted by Eq. (1) that the log-transformed expression of Cobb-Douglas function as its typical generalization form for its solution (Labini 1995; Vilcu 2011) is adopted in this work to interpret the short-term change of the passenger flow into a certain entrance of a metro station in different time intervals:

$$\ln(Q_i) = \alpha_0 + \alpha_1 \ln(Q_{i-1}) + \alpha_2 T + \sum_{j=1}^n \beta_j \ln(X_{i-\Delta_j}^j) + \varepsilon_i, \quad (1)$$

where: Q_i represents the volume of the passengers entering the station in the time interval i ($i \in N$); Q_{i-1} stands for the volume of the passengers entering the station in the time interval $i-1$; T is the time-trend variable; $X_{i-\Delta_j}^j$ represents the value of the j -th related factor in

the time interval $i - \Delta_j$ ($\Delta_j \in N$ and $\Delta_j < i$); n denotes the number of all the related factors; ε_i is the error term following the prior probability distribution of $N(0, \sigma^2)$; $\alpha_0, \alpha_1, \alpha_2$ and β_j ($j = 1, 2, \dots, n$) are the parameters.

With Gibbs sampling (Arnold 1993), the hierarchical Bayesian approach (Gelman et al. 2013) based on Markov Chain Monte Carlo (Brooks et al. 2011) is applied in this work to calibrate the parameters in Eq. (1). This approach assumes the prior probability distributions of the parameters and yields a chain for making their point or interval estimates according to the successive sampling from their posterior probability distributions (Chikaraishi et al. 2010). The joint posterior probability distribution of all the parameters in Eq. (1) is explained by Eq. (2):

$$\begin{aligned} & \pi(\alpha, \beta; \Gamma; \sigma | Q, X) \propto \\ & \prod_i g(Q_i, X_{i-\Delta_1}^1, \dots, X_{i-\Delta_n}^n | \alpha, \beta; \sigma) \\ & P(\alpha, \beta | \Gamma) P(\Gamma) P(\sigma), \end{aligned} \quad (2)$$

where: α stands for α_0, α_1 and α_2 ; β denotes $\beta_1, \beta_2, \dots, \beta_n$; Q represents the passenger volumes; X denotes the values of the related factors; Γ is the variance-covariance matrix of the multivariate normal distribution followed by the random components of $\alpha_0, \alpha_1, \alpha_2, \beta_1, \beta_2, \dots, \beta_n$; σ represents the standard deviation of the prior probability distribution of ε_i ; $g(\cdot)$ is the likelihood function; $P(\Gamma)$ represents the prior probability distribution of Γ ; $P(\sigma)$ stands for the prior probability distribution of σ .

Because of insufficient prior information, the assumption of a non-informative prior probability distribution – i.e. a non-committal prior probability distribution, e.g. typically a rectangular distribution, over the feasible set of values (Upton; Cook 2008) – is initially made in this research for each of the parameters in Eq. (1). Moreover, the prior probabilities of Γ and σ are assumed here to respectively follow inverted Wishart distribution and inverted Gamma distribution. As interpreted by Eq. (3), the posterior distributions of the estimated parameters can be improved with new inputs by sequential Bayesian updating (Ntzoufras 2009) on the basis of Bayes' Rule (Stone 2013):

$$\begin{aligned} & \pi(\alpha, \beta; \Gamma; \sigma | Q, X, Q_{i+1}, X_{i+1-\Delta_j}^j) \propto \\ & g(Q_{i+1}, X_{i+1-\Delta_1}^1, \dots, X_{i+1-\Delta_n}^n | \alpha, \beta; \sigma) \\ & \pi(\alpha, \beta; \Gamma; \sigma | Q, X). \end{aligned} \quad (3)$$

Based on the data surveyed on 27 October 2013, the estimations to all the parameters in Eq. (1) are iteratively made according to the hierarchical Bayesian approach with taking the posterior distributions in last calibration as the prior distributions in the next iteration. In order to ensure effective convergences, the number of the iterative calibrations of the parameters is set to be 1 million times in this study according to the tests of convergence computations. If the calibrations have not converged after 1 million times of the iterations, another 1 million times of the iterative calibrations will be con-

tinually made until the convergence of the estimations. In addition, on the basis of the sequential Bayesian updating, the posterior distributions of the estimates are updated in each of the iterative calibrations with the data from a supplementary survey made on 15 June 2014, for certain representative daily operation time of XS in view of the effect of climate change and so on. After the iterative estimations, 1000 sets of the updated posterior probability distributions of the parameters are successively extracted randomly from every 100 sets of the outcomes obtained in the last 100 thousand calibrations. These extracted updated posterior distributions are used to not only identify the convergence of the iterative estimations by Geweke’s tests (Ntzoufras 2009) but also compute the final values of the parameters by Eq. (4). The whole process of the iterative estimations to the parameters is shown in Fig. 4.

$$\gamma_k = \frac{1}{m} \sum_{l=1}^m \gamma_k^l, \tag{4}$$

where: γ_k is the final value of the k -th parameter in Eq. (1); m represents the number of all the extracted iterations; γ_k^l stands for a random value of the k -th estimated parameter following its posterior distribution updated in the l -th extracted iteration.

The final estimation results of Eq. (1) for the predictions of the passenger flows into XS through F1 and B are provided in Table 1 and Table 2 to explain the trends of their respective increases and decreases with the successive time intervals. As for the forecast of a passenger flow into XS in its increase time trend, the related factors are successively the amounts of the people utilizing the escalators rolling downstairs from the second floors of Juntai and Hanguang, and the people walking towards

XS from RI. By contrast, the prediction of the volume of the passengers entering XS in its decrease trend with time passed focuses only on the numbers of the people on the escalators rolling downstairs from the second floor of Juntai and the people from RI in succession. These different factors work independently. It is shown in both these two tables that the estimated values of the parameters are all logically reasonable and each of their Geweke’s test values, i.e. the Z-values, is between -2.00 and 2.00 , which proves the effective convergences of the estimations. In addition, the model estimation accuracies represented by the R-square values for F1 and B are both satisfying enough.

Table 1. Model estimation results for the increased time-trend

Parameters	F1		B	
	Estimated values	Z-values	Estimated values	Z-values
α_0	0.52	-0.28	0.58	-0.09
α_1	0.67	-0.81	0.57	-0.28
α_2	0.01	0.66	0.01	0.44
β_1	0.11	-0.20	0.11	-1.92
β_2	0.07	0.62	0.08	0.76
β_3	0.19	0.70	0.29	0.24
σ	0.19	0.21	0.19	0.21
R^2	0.93		0.93	

Table 2. Model estimation results for the decreased time-trend

Parameters	F1		B	
	Estimated values	Z-values	Estimated values	Z-values
α_0	2.70	-0.40	2.69	-0.22
α_1	0.29	-1.08	0.23	-0.41
α_2	-0.01	0.59	-0.01	-0.17
β_1	0.25	1.85	0.24	0.18
β_2	0.33	-0.57	0.33	0.65
σ	0.13	1.49	0.13	1.49
R^2	0.86		0.93	

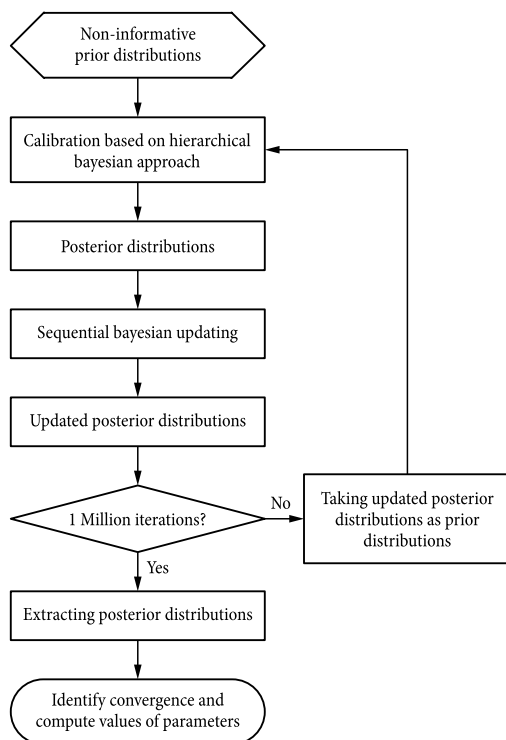


Fig. 4. Flowchart of the estimation process

3. Case Analysis

It is clarified in Fig. 1 that the maximum intensities of the passenger flows per 5 minutes into B and F1 on Sunday are stably more than respectively 350 and 250 and take shape at around 16:00 and 18:00 correspondingly. Given additional sharp increases of the passenger flows in such time because of abrupt contributions from nearby important buildings (i.e. Juntai, Hanguang and RI), the operation safety of XS will be under serious threat if countermeasures of XS are not fully taken in time. In the light of the field survey, the biggest numbers of the people using the escalators going downstairs from the second floors of Juntai and Hanguang, and the people walking towards XS from RI are able to in extreme correspondingly exceed 740, 740 and 1800 in 5 minutes. Taking the lower limits of the maximum stable passen-

ger flows (i.e. 350 and 250) explained above as the passenger flows in last time interval, the random coefficient model estimated in Section 2 is utilized here to analyze the extreme effect of the related factors on the short-term passenger flows into B and F1 in subsequent time intervals. The designed service capacities of some facilities and equipments adopted in a metro station in China under ideal conditions are introduced in Table 3 according to the works of Mao (2011) and BUEDRI (2003).

Table 3. Service capacities of some facilities and equipment

Facilities and equipment		Service capacity (passengers per hour)
Ticket checking machine	door-style	1800–2100
	three-pole-style	1500–1800
Passage (1 meter wide)	unidirectional	5000
	bidirectional	4000
Stairway (1 meter wide)	unidirectional upstairs	3700
	unidirectional downstairs	4200
	bidirectional	3200

It is found that the intensities of the passenger flows into XS through B and F1 are possible within only about 10 minutes to become close to 6600 and over 4400 per hour in extreme for certain operation time of XS after the sudden volume increases of the related factors interpreted previously. It can be proved in computation based on the capacity values provided in Table 3 that even though only less than 50% of the service capacities of the passages, stairways and ticket checking machines can be truly utilized in reality (Yang 2010), these facilities and equipment in corresponding areas inside XS now are respectively as wide and many as enough to fully serve these two passenger flows. Nevertheless, in this case, the passenger flows will be gathered on the platforms of XS very quickly. This will make the security of the passengers waiting for trains impossible to be guaranteed with full certainty due to the very limited areas of the platforms especially when the headways of the trains successively arriving at and departing from XS are not short enough to cope with the sharp increases of the passenger flows into XS. As a result, slowing down the speeds of the passenger flows into XS with effective countermeasures such as ordinarily extending the walking distances of the routes to the entrances of the station by, for example, setting temporary fences to certain distance from the entrances is indispensable at such time. In order to take the countermeasures in time before especially an extreme passenger flow attack, the dynamic information of the factors associated with the passenger flows into a station is also very essential to be promptly transferred from, for instance, busy department stores in neighborhood to the management authority of the station under the premise of the standby of its spare manpower, mobile facilities and so on.

Conclusions

Based on the representatively dynamically abrupt changes of the passenger flows into Xidan Station (XS) in different time intervals of its daily operation on Sunday, a random coefficient model is newly developed in this research to forecast the short-term changes of the passenger flows into an urban rail transit station. The parameters of this model are iteratively calibrated with hierarchical Bayesian approach and updated by sequential Bayesian updating in each of the iterative calibrations. It has been confirmed that the estimations to the parameters are able to effectively get their reasonable convergences with satisfactory accuracies. The case study with the application of the proposed random coefficient model shows that the newly developed model is capable of rationally predicting the exact short-term changes of the passenger flows into an urban rail transit station. This enables the rail transit station to quickly act moderately before the hits of sudden volume increases of the passenger flows into it if the station has adequate backup manpower, facilities and so on in particular for a special day with a huge passenger volume pressure. Furthermore, it is also suggested that the instant linkage of the information about different factors taking effect on the passenger flows into an urban rail transit station with nearby important buildings, squares, crossroads, etc. which are easily able to cause outbursts of the passenger flows is very essential. This enables the station to take appropriate actions in advance for ensuring its operation safety facing up to the threat of sharp increases of the passenger flows.

Only a representative metro station, i.e. XS, in Beijing has been analyzed in this research. Moreover, due to the limited amount of the survey data, the types of the related factors on which this work focuses for the studied station are not rich at this stage. Therefore, many other different kinds of typical urban rail transit stations in the world need to be studied with much more passenger flow data surveyed in future research to further valid the results of this research. Furthermore, the interactive effect of all the passenger flows into different entrances of an urban rail transit station at the same time on the safety of the passengers on the platforms of this station shall be analyzed as well. In addition, the comparative analyzes of the effect of the random coefficient model proposed in this study and other forecasting techniques for the predictions of short-term changes of various kinds of travel demands also ought to be made in the future to enrich this work.

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