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Analytical problematical paper

DETERMINATION OF THE ANGLES OF PRESSURE IN THE SYSTEM OF LIMITATION BY OPTIMISATION OF LINKAGES

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Abstract. In the present paper a method of analytical determination of reactions and angles of pressure is considered with the four-bar linkage as an example. The determined sequence for the expression of acceleration of the points and angle acceleration of the links is presented. Further the power analysis for Assur's group is made. The equations of normal components of reactions are expressed as straight lines passing across given point in the given direction. The equations of the full reactions in the hinges are expressed as straight lines passing across two given points. After that the angles of pressure are expressed as the angles between directions of the speed and reactions.

Keywords: speed, acceleration, force, equation, angle of pressure.

1. Introduction

The aim of the research is to examine of the angles of pressure as the conditions for an algorithm of optimization of the linkages. In earlier publications [1, 2] analytical determination of the speeds and acceleration was described when the planes were considered as illustrations without construction and dimensions. In the present paper the same way of determination of reactions and angles of pressure is given. For example, the four-bar linkage is considered, scheme presented in Fig 1.

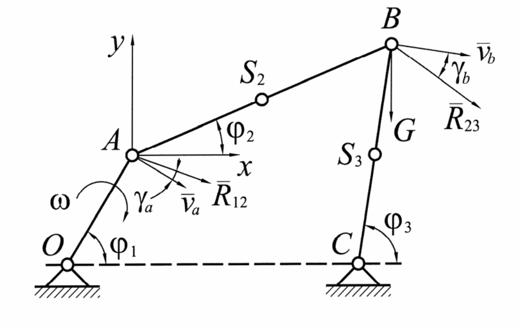

Fig 1. Four-bar linkage

2. Mathematical model

The angle of pressure is the angle between the direction of the force put to supporting link at any point and the direction of the speed of that point [3].

The equations of the speed of the points A, B are expressed as straight lines in standard form.

Point A is taken as the start of coordinates (Fig 1).

For the speed:

$$\bar{v}_A: \; y_A = -\frac{x_A}{\text{tg}\varphi_1}.$$

For the speed:

$$\bar{v}_B: \; y_B = l_2 \sin\varphi_2 - \frac{x_B - l_2\cos\varphi_2}{\text{tg}\varphi_3}.$$

Let us put the equations to standard form:

$$A_1 x_A + B_1 y_A + C_1 = 0, \quad (1)$$

where $A_1 = 1$; $B_1 = \text{tg}\varphi_1$; $C_1 = 0$.

Similarly to $\bar{v}_B$:

$$A_2 x_B + B_2 y_B + C_2 = 0\,, \qquad (2)$$

where

$A_2 = 1$; $B_2 = \mathrm{tg}\varphi_3$;

$C_2 = -l_2(\cos\varphi_2 + \sin\varphi_2 \cdot \mathrm{tg}\varphi_3)$.

The accelerations of the points are determined, so as to express the acceleration of the centres of masses and the angle acceleration of the links in the following sequence [1]:

1) coordinates of the points of the plane of acceleration known by quantity and direction are determined;
2) the equations of the vectors known only by direction (across given point in the given direction) are expressed;
3) coordinates of the point of intersection of the last vectors are determined;
4) values of acceleration of the centres of masses and angle acceleration of links are determined.

Then the forces of inertia and their moments are determined. Further Assur's group is considered (Fig 2). Tangential components of the reactions in the extreme hinges of Assur's group are calculated.

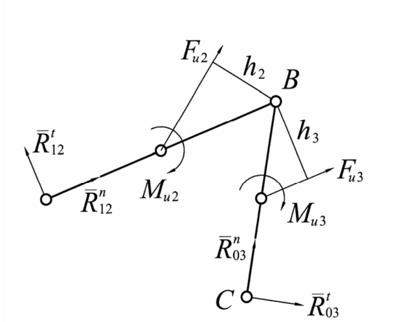

Fig 2. Assur's group

For the *link 3*: from the equation $\sum M_B = 0$, that is $\bar{R}^t_{03} l_3 + F_{u_3} h_3 - M_{u_3} = 0$ the $\bar{R}^t_{03}$ is determined.

Similarly for the *link 2*: from the equation $\sum M_B = 0$ the $\bar{R}^t_{12}$ is determined.

Coordinates of beginnings and ends of vectors known by quantity and direction are determined.

After that a chain of vectors is completed by addition $\bar{R}^t_{12}$ to the beginning of the chain and $\bar{R}^t_{03}$ to the end.

The equations of normal components of reactions as straight lines passing across given point in the given direction are expressed [4].

For the vector $\bar{R}^n_{12}$:

$$y - y_1 - k_1(x - x_1) = 0\,. \qquad (3)$$

For the vector $\bar{R}^n_{03}$:

$$y - y_2 - k_2(x - x_2) = 0\,. \qquad (4)$$

In the equations (3), (4) x_2, y_2 are coordinates of the end of vector $\bar{R}^t_{03}$; x_1, y_1 are coordinates of the beginning of $\bar{R}^t_{12}$; $k_1 = \mathrm{tg}\varphi_2$; $k_2 = \mathrm{tg}\varphi_3$.

After leading equations (3), (4) to standard form, coordinates of the point of intersection of these lines are determined:

$$x_0 = \frac{\begin{vmatrix} B_3 C_3 \\ B_4 C_4 \end{vmatrix}}{\begin{vmatrix} A_3 B_3 \\ A_4 B_4 \end{vmatrix}}\,; \quad y_0 = \frac{\begin{vmatrix} C_3 A_3 \\ C_4 A_4 \end{vmatrix}}{\begin{vmatrix} A_3 B_3 \\ A_4 B_4 \end{vmatrix}}\,, \qquad (5)$$

where $A_{3,4} = -k_{1,2}$; $B_{3,4} = 1$; $C_{3,4} = k_{1,2} x_{1,2} - y_{1,2}$.

By the vector equation of the plane of forces for one link of the group, vector $\bar{R}_{23}$ in a middle hinge B in Assur's group is determined.

The equations of reactions in the hinges A and B are expressed as the straight lines passing across 2 given points (x_0, y_0 – coordinates of one of them are the common end of $\bar{R}_{12}$ and $\bar{R}_{23}$):

$$(y - y_0)(x_{3,4} - x_0) - (y_{3,4} - y_0)(x - x_0) = 0\,, \quad (6)$$

where x_3, y_3 are coordinates of the beginning $\bar{R}_{12}$; x_4, y_4 are coordinates of the beginning $\bar{R}_{23}$.

3. Conclusion

Thereafter the angles of pressure are determined as the angles between directions of speed and reactions [4]:

$$\mathrm{tg}\gamma_{A,B} = \frac{A_{1,2} B_{5,6} - A_{5,6} B_{1,2}}{A_{1,2} A_{5,6} + B_{1,2} B_{5,6}}\,, \qquad (7)$$

where $\gamma_{A,B} \le [\gamma]$; $A_{1,2}$, $B_{1,2}$ are coefficients of the equations of speed in standard form; $A_{5,6}$, $B_{5,6}$ are coefficients of the equations of reactions in standard form.

In the case of arising of possible angles of pressure corresponding combinations of links are not taken into account.

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