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DEPENDENCE OF MULTI-CRITERIA EVALUATION RESULT ON CHOICE OF PREFERENCE FUNCTIONS AND THEIR PARAMETERS

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Abstract. A considerable usage increase of multicriteria methods is recently observed in the area of quantitative analysis of social or economical phenomena. The PROMETHEE methods are discerned from other multi-criteria methods by depth of their intrinsic logic and by using preference functions, which make up a foundation of the methods. Shapes of functions and their parameters are chosen by decision-makers thus exerting clear advantages and features of the methods. This paper reveals influence of the choice of preference functions and their parameters on the outcome of evaluation. Along with already recently described by the authors PROMETHEE I method the other PROMETHEE II method is described and examples of its application are provided. New types of preference functions were proposed.

Keywords: preference functions, parameters of preference functions, multi-criteria evaluation, PROMETHEE methods.

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1. Introduction

Reality often raises the task of evaluation of several possible alternatives and outlining them in the order of preference. Such task could be selection of the best alternative among investment projects, evaluation of different regions of a country or rate of development of different countries, etc.

A considerable usage increase of multi-criteria methods is recently observed in quantitative analysis of social or economical phenomena or other complex processes (Brauers, Zavads-

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kas 2006; Brauers *et al.* 2007; Figueira *et al.* 2005; Hui *et al.* 2009; Maskeliūnaitė *et al.* 2009; Plebankiewicz 2009; Podvezko 2009; Turskis *et al.* 2009; Ulubeyli, Kazaz 2009; Ustinovichius *et al.* 2007; Zavadskas, Antuchevičienė 2006; Zavadskas *et al.* 2008a, b, 2009).

The range of the PROMETHEE (Preference Ranking Organisation Method for Enrichment Evaluation) group of methods is wide: from the PROMETHEE I method indicating the best alternative among the ones in question, the PROMETHEE II (full classification method), which is ranging alternatives in respect of desired objectives, up to the PROMETHEE VI method, which yields an indication if the problem is hard or soft, and the visual model GAIA (Brans, Mareschal 1992, 1994, 1996, 2005).

The PROMETHEE methods are well-known and are often being used. Bibliography comprises hundreds of publications (Brans, Mareschal 2005; Behzadian *et al.* 2010). PROMETHEE methods were used in many different areas, from logistics to health service (Behzadian *et al.* 2010; Brans, Mareschal 2005). Lithuania is at the initial stage of using the methods (Nowak 2005; Podvezko, Podviezko 2009).

PROMETHEE methods comprise criteria values of chosen indices and their weights in more sophisticated way by using preference functions with few parameters. Preference function shapes and their parameters are chosen by responsible persons of the evaluation, decision-makers or qualified experts. In addition to already existing, new types of preference functions were proposed in this paper, with intention of widening the range of choice for decision-makers and evaluation experts.

The goal of this paper is to extend and deepen study of this method, to describe the algorithm of the PROMETHEE II method, to apply this method to obtain outranking relationship of alternatives, to add some knacks to this method, to broaden the scope of users and to demonstrate dependence of results of evaluation on the choice of shapes of preference functions and their parameters.

2. The brief description of the algorithm of the PROMETHEE methods

We will briefly recall the algorithm of the PROMETHEE methods (Brans, Mareschal 2005; Podvezko, Podviezko 2009). The core of this method is the same as in other multi-criteria methods. The method uses criteria value matrix of statistical data or experts' assessment data $R = \|r_{ij}\|$ characterising objects being evaluated and weights of criteria $\left(\sum_{i=1}^{m} \omega_i = 1\right)$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$, where m is the number of criteria, n is the number of evaluated objects or alternatives. Every criterion must be defined to be maximising or minimising. Maximum values of maximising criteria are considered to be the best as minimum values of minimising criteria. Multi-criteria methods usually use normalised criteria values $\tilde{r}_{ij}$ and weights ω_i. A good example is SAW (Simple Additive Weighting) method, which suggests formula for calculation criteria of evaluation (Hwang, Yoon 1981; Ginevičius, Podvezko 2008a, b, c, 2009; Ginevičius *et al.* 2008a, b; Jakimavičius, Burinskienė 2009):

$$S_j = \sum_{i=1}^{m} \omega_i \tilde{r}_{ij}, \tag{1}$$

PROMETHEE methods use values of so-called preference functions $p(d)$ instead of normalised values of criteria $\tilde{r}_{ij}$.

The range of values of preference functions falls between zero and one. Values of the functions reveal the level of preference of one alternative over another. Shapes of functions depend on boundary parameters q and s, which are chosen by a decision-maker for each criterion i, namely q_i for the lower and s_i for the upper boundary of the argument thus making two alternatives A_j and A_k indifferent in respect of the criteria R_i when the difference between values of criteria r_{ij} and r_{ik} for these alternatives $d_i(A_j, A_k) = r_{ij} - r_{ik}$ is smaller than the boundary parameter q_i and thus making the alternative A_j of the strict preference in favour of the alternative A_k when the difference between criteria values r_{ij} and r_{ik} for these alternatives $d_i(A_j, A_k) = r_{ij} - r_{ik}$ is greater than the boundary parameter s_i. When the difference falls between q_i and s_i preference criterion of the alternative A_j in respect of the alternative A_k varies between zero and one.

PROMETHEE methods suggest the following formula for calculation the aggregated preference index $\pi(A_j, A_k)$ of the alternative A_j in respect of the alternative A_k:

$$\pi(A_j, A_k) = \sum_{i=1}^{m} \omega_i \, p_t\left(d_i(A_j, A_k)\right), \qquad (2)$$

where ω_i is the weight of the i-th criterion $\left(\sum_{i=1}^{m} \omega_i = 1\right)$; $d_i(A_j, A_k) = r_{ij} - r_{ik}$ is the difference between values r_{ij} and r_{ik} of the criterion R_i for the alternatives A_j and A_k; $p_t(d) = p_t(d_i(A_j, A_k))$ is the t-th preference function chosen by a decision-maker for the i-th criterion from the set of available preference functions.

The PROMETHEE method adds all positive preference indices and thus the positive outranking flow is obtained

$$F_j^+ = \sum_{k=1}^{n} \pi\left(A_j, A_k\right) \quad \left(j = 1, 2, \ldots, n\right) \qquad (3)$$

and all negative preference indices to have the negative outranking flow

$$F_j^- = \sum_{k=1}^{n} \pi\left(A_k, A_j\right) \quad \left(j = 1, 2, \ldots, n\right). \qquad (4)$$

The PROMETHEE I method reveals mutual outranking relationship between alternatives A_j and A_k by summing all "outgoing" and "incoming" outranking indices with respective positive or negative sign. Possible outcomes are denoted as P^+, P^-, I^+, I^- (Brans, Mareschal 2005; Podvezko, Podviezko 2009).

Thus, the alternative A_j is outranking the alternative A_k (or $A_j P A_k$), if $F^+(A_j) > F^+(A_k)$ (or $A_j P^+ A_k$) and $F^-(A_j) < F^-(A_k)$ (or $A_j P^- A_k$). The same holds if $A_j P^+ A_k$ and $A_j I^- A_k$ ($F^-(A_j) = F^-(A_k)$), or in case if $A_j I^+ A_k$ and $A_j P^- A_k$.

Similarly, indifference and incomparability of alternatives A_j and A_k are described.

The PROMETHEE II method uses the idea of the PROMETHEE I method. But in addition it lists all evaluated alternatives in accordance with the level of their attractiveness, which is measured by the value of the difference (the net outranking flow) $F_j = F_j^+ - F_j^-$. The biggest difference between all positive ("outgoing") preference indices F_j^+ and negative ("incoming") preference indices F_j^- ($j = 1, 2, ..., n$) corresponds to the best alternative. The PROMETHEE II method is ranging alternatives in decreasing order in respect of values F_j.

In contrast to the PROMETHEE II method, the PROMETHEE I method was designed to indicate only the best alternative, for which the number of worse alternatives in terms of preference is the highest.

3. Preference functions and their features

As was already mentioned, the argument d of preference function $p(d)$ is the difference of criteria values. More precisely, for the i-th criterion for alternatives A_j and A_k, we have $d_i(A_j, A_k) = r_{ij} - r_{ik}$, where r_{ij} and r_{ik} are values for the criterion i for mentioned alternatives. In spite of the fact that preference functions are of similar purpose as normalised values of data in other multi-criteria methods, their features and practical realisation are much more profound. We outline main features of preference functions:

- values of preference functions are falling to the interval from zero to one: $0 \le p(d) \le 1$;
- preference functions were projected to be functions representing maximising criteria by normalised values; the higher is value of the function $p(d)$, the higher is preference of the alternative;
- preference function $p(d)$ value equals to zero when the difference d is smaller than the boundary value q: $p(d) = 0$ when $d \le q$ (in some cases the boundary value q is not set and it is implied that $q = 0$);
- in case when the upper boundary value s of the difference of values is set, then $p(d) = 1$ whenever $d \ge s$ (there are cases when the upper boundary value s is not set and $\lim_{d \to \infty} p(d) = 1$).

There are known six preference functions $p(d)$ (Brans, Mareschal 2005; Podvezko, Podviezko 2009), although some new preference functions will be proposed in this paper.

1. The so-called *usual* preference function could be used only in cases, when the decision-maker cannot allocate importance for the differences between criteria values and only seems to know the formula "the more the better". This function does not depend on parameters q and s. In other words, the lower and the upper boundary values are not set for this type of preference function. This function could be proposed only in such cases when it is only important that the difference $d_i(A_j, A_k) = r_{ij} - r_{ik}$ between values r_{ij} and r_{ik} is positive ($p(d) = 1$) or negative ($p(d) = 0$) and the value of the difference does not matter. For example, one job offer is preferred over another if offered salary is higher without assigning any importance to the difference; it is important if distance to the office is higher or smaller; if interest rate offered by banks for term deposits is higher or smaller; if length of work experience between two candidates for a job is higher or smaller; if gasoline price at two gas stations is higher or smaller; if price between two investment projects is higher or smaller; if one candidate for a job knows more languages than another; if processor speed of one computer for sale is higher or lower than another's, etc.

We emphasise the fact that preference function is used in simultaneous pairwise evaluation by all m criteria. For example, the multi-criteria evaluation of candidates for a job offer will be conducted by simultaneous comparison of their length of work experience, level of education, knowledge of foreign languages, age, etc. By the other hand, the candidate will himself simultaneously compare salary, perspective, colleagues, distance to the office, office space, fringe benefits, etc.

The analytical expression and the shape of the first *usual* function are given on Fig. 1.

$$p(d) = \begin{cases} 0, \text{ when } d \le 0 \\ 1, \text{ when } d > 0 \end{cases}$$

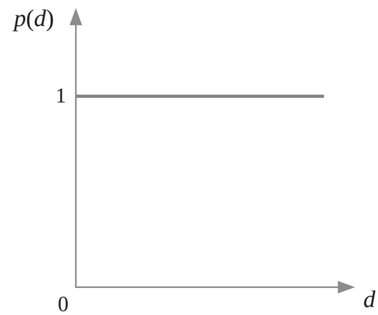

Fig. 1. *Usual* preference function and its graph

2. The second *U-shape* preference function differs from the *usual* one by setting the lower boundary value q (here it is identical to the upper boundary value s), starting from which the difference of values of applied criterion is considered to induce the strict preference of one alternative over another. So, when the difference d is higher than q, value of the preference function equals to one and $p(d) = 0$ when $d \le q$.

The analytical expression and the shape of the second *U-shape* preference function are given on Fig. 2.

$$p(d) = \begin{cases} 0, \text{ when } d \le q \\ 1, \text{ when } d > q \end{cases}$$

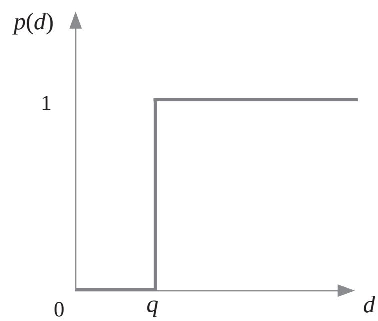

Fig. 2. *U-shape* preference function and its graph

This function has a higher practical importance comparing with the first *usual* preference function. We can easily adopt the above mentioned examples to fit them to the case of *U-shape* preference function. The new job will have strict preference ($p(d) = 1$) over another only in case if salary differs by no less than 100 euros ($q = 100$) and is of no importance to the employee ($p(d) = 0$), if an offered salary exceeds by less than 100 euros comparing to another offer. A bank's offer will be of interest in case if interest rate for term deposits exceeds 1% comparing to another bank's offer ($q = 1$); a candidate will be of interest in case his work experience exceeds work experience of another candidate by three years ($q = 3$) or he correctly answers at least three test questions more than another candidate and so on.

3. The third *V-shape* (or *linear preference*) preference function differs from the previous one in the interval from zero to s, where the link between the point of indifference of alternatives ($p(d) = 0$), no preference of one alternative over another) and the point of strict preference of one alternative over another ($p(d) = 1$) is not of a shape of a shift, but is linear. Another difference is by setting the upper boundary parameter s, from which one alternative has strict preference over another instead of the lower boundary parameter q, until which both alternatives are indifferent.

The analytical expression and the shape of the third *V-shape* preference function are given on Fig. 3.

$$p(d) = \begin{cases} 0, & \text{when } d \le 0 \\ \dfrac{d}{s}, & \text{when } 0 < d \le s \\ 1, & \text{when } d > s \end{cases}$$

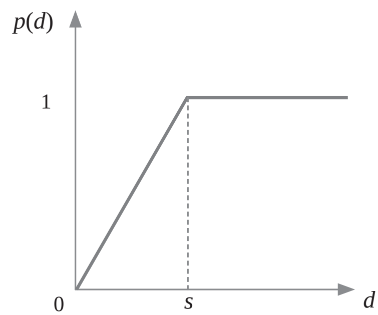

Fig. 3. *V-shape* preference function and its graph

Again, we can apply previous examples to this case of preference function by their slight modifying. Now, a job offer will have a strict preference over another in case of salary difference of 100 euros or more, is of no interest in case a lower salary is offered ($p(d) = 0$, when d is negative) and is of some gradually increasing interest in case the difference is up to 100 euros ($0 < d \le 100$). Preference function value is then expressed by the formula: $p(d) = \dfrac{d}{100}$. Other examples could be easily modified in the similar way.

4. The fourth preference function is called *level* preference function. It depends on two parameters p and q, thus both boundary values are set: the indifference boundary q and the strict preference boundary s. So, in case if the difference d of values of two alternatives is not greater than q, then the alternatives are indifferent ($p(d) = 0$); when the difference d is greater than s, then one alternative has the strict preference over another and whenever the difference d falls between q and s, or $d \in [q, s]$ then value of the preference function equals to 0.5. In this case one alternative has a medium preference over another.

The analytical expression and the shape of the fourth *level* preference function are given on Fig. 4.

$$p(d)=\begin{cases}0, & \text{when } d \le q\\ 0.5, & \text{when } q < d \le s\\ 1, & \text{when } d > s\end{cases}$$

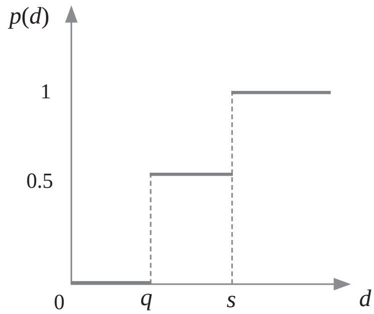

Fig. 4. *Level* preference function and its graph

For example, a candidate for a job will have no advantage if he knows less foreign languages than another candidate ($p(d) = 0$, d is negative) some advantage in case if he knows one language more than another candidate ($p(d) = 0.5$), and will have strict preference over another candidate in case he knows two more languages than another candidate ($p(d) = 1$). A similar preference function but with more step gradations could be used in case of more discreet options. It approximates the linear function as the number of gradations increases.

5. The fifth *V-shape with indifference* preference function (as well as *level* preference function) has both parameters q and s, which set boundaries of indifference and strict preference. But when the difference criteria values of two alternatives falls into the interval from q to s, or $d \in [q, s]$, the preference function uniformly linearly increases from zero to one in accordance with the formula $\frac{d-q}{s-q}$ and its value indicates the level of preference of one alternative over another. In the case when $q = 0$ this function becomes the third *V-shape* preference function.

Another example described above again could be easily transformed to this particular case. An employee will be indifferent if salary between two job offers differs by less than 100 euros ($p(d) = 0$). The new job will be of strict preference in case if salary in the new job offer exceeds 500 euros ($p(d) = 1$) and the new job will be of some preference over another in case if salary in the new job offer exceeds by a number between 100 and 500 euros; the level of preference is calculated by the formula $p(d)=\frac{d-100}{500-100}=\frac{d-100}{400}$.

Other examples can be easily transformed similarly.

The analytical expression and the shape of the fifth *V-shape with indifference* preference function are given on Fig. 5.

This function is the most valuable and it attracts the largest number of theoretical and practical applications for evaluations carried out by PROMETHEE methods.

6. The sixth *Gaussian* preference function is used in case the initial statistical data is consisting of random values with the normal distribution. Preference at low differences of criteria values increases slowly by increase of d, starting from zero. The same applies also at large differences $d_i(A_j, A_k)$ of criteria values; the preference function in this case is gradually approaching one never reaching this value. This function requires a parameter σ of stand-

ard deviation of given random data, and is increasing most rapidly at values of differences *d* close to σ.

$$p(d)=\begin{cases}0, & \text{when } d\le q\\ \dfrac{d-q}{s-q}, & \text{when } q<d\le s\\ 1, & \text{when } d>s\end{cases}$$

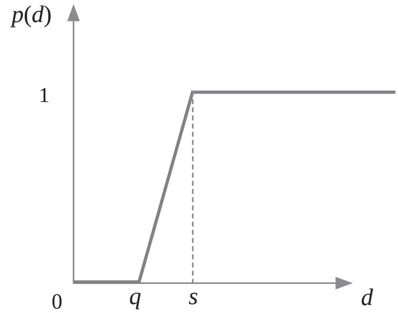

Fig. 5. *V-shape with indifference* preference function and its graph

The analytical expression and the shape of the fourth *Gaussian* preference function are given on Fig. 6.

$$p(d)=\begin{cases}0, & \text{when } d\le 0\\ 1-\exp(-\dfrac{d^2}{2\sigma^2}), & \text{when } d>0\end{cases}$$

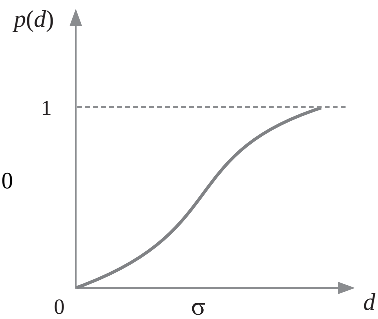

Fig. 6. *Gaussian* preference function and its graph

We propose several new preference functions.

7. *Multistage* preference function. Some alternatives can only have discreet criteria values. Very often, they are natural positive numbers. Consider the number of spoken languages, number of children in a family, number of stock in farms, number of shops in a supermarket chain in a town, number of ATM machines possessed by a bank. In all such cases, differences of criteria values are discrete or are natural numbers (positive and negative). Quite interesting is the case, when criteria values are real numbers, like amounts in euros, but criteria of preference should be expressed in natural numbers. For example, consider the fact that the GDP plan is usually revealed to the public and will be perceived in billions, while projection versions are given in real numbers. Consequently, evaluation of the plan or its outcome in public is going to be in integer billions, not in real numbers. In addition, consider evaluation of bank performance. Precise data is produced in real numbers while evaluation is going to be made and discussed in millions. Price for a large possession is given in real numbers while

perception of the price is going to be in thousands. These examples show how important might be the multistage preference function in order to match expert's perception of the criterion. The fourth *level* function with its only values 0, 0.5, 1 is too rough to deal with all mentioned cases.

For integer criteria values we must have the largest difference $d = s$, where s is integer number. In case it is not available, take $s = \max_j r_{ij}$ or any lower value, which sets an expert.

For real criteria values, the analytical expression and the shape of the seventh new *multistage* preference function are given on Fig. 7.

$$p(d) = \begin{cases} 0, \text{ when } d \le 0 \\ \frac{1}{s}, \text{ when } 0 < d \le 1 \\ \frac{2}{s}, \text{ when } 1 < d \le 2 \\ \dots \\ \frac{s-1}{s}, \text{ when } s-2 < d \le s-1 \\ 1, \text{ when } s-1 < d \le s \le \max_j r_{ij} \end{cases}$$

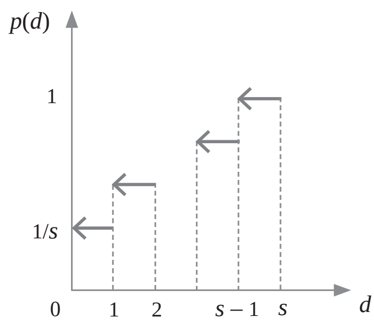

Fig. 7. *Multistage* preference function and its graph for real criteria values

In case criteria values are discrete, the function can be defined in a different way. The analytical expression of the *multistage* preference function for discrete criteria values is given in formula (5):

$$p(d) = \begin{cases} 0, \text{ when } d \le 0 \\ \frac{1}{s}, \text{ when } d = 1 \\ \frac{2}{s}, \text{ when } d = 2 \\ \dots \\ \frac{s-1}{s}, \text{ when } d = s-1 \\ 1, \text{ when } d = s \le \max_j r_{ij} \end{cases} \tag{5}$$

8. The eighth *C-shape* preference function is rapidly increasing at low differences of criteria values $d_i(A_j, A_k)$ by increase of d, starting from zero. The higher become values of difference d, the smaller is relative increase of preference function. This function is somewhat similar to the *linear priority function*, although is sensitive to even large differences of criteria values and induces more relative sensitivity at low differences d.

The analytical expression and the shape of the seventh *C-shape* preference function are given on Fig. 8.

$$p(d)=\begin{cases}0, & \text{when } d \le 0\\ \sqrt{\dfrac{d}{s}}, & \text{when } 0 < d \le s\\ 1, & \text{when } d > s\end{cases}$$

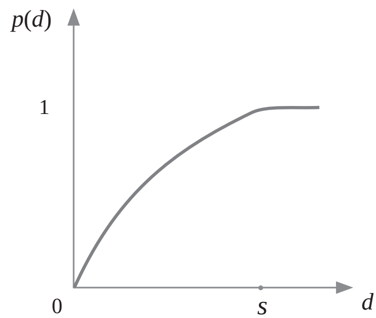

Fig. 8. *C-shape* preference function and its graph

This function could be used instead of the third *V-shape* preference function; it fits better for such cases when small differences between two criteria values induce more relative importance than large differences. A good illustration is again job-searching, when small increases of salary are usually of more *relative* practical value than high increases.

We also propose some other preference function: $p(d)=\sqrt[3]{\dfrac{d}{s}}$ (its shape looks similar to the one shown on the 8-th graph), $p(d)=\dfrac{2}{\pi}\,arctg\ d$ (its shape looks similar to the shape of the 6th preference function, but is applicable for non-statistical data).

4. Dependence of evaluation result on choice of preference function types and their parameters

Dependence of evaluation result will be illustrated by the example of growing of economies of the Baltic States and Poland for the year of 2003. Calculations were made using different multi-criteria methods (Ginevičius *et al.* 2006). A solution having used the PROMETHEE I method was already demonstrated (Podvezko, Podviezko 2009). Statistical data is given in Table 1.

Table 1. Criteria values of economical growth of different countries

	Criteria	Types of criteria	Estonia	Latvia	Lithuania	Poland
1	Annual growth of the GDP, %	max	5.1	7.5	9.7	3.8
2	Annual growth of production, %	max	9.8	6.5	16.1	8.4
3	Average annual salary in euros, %	max	430	298	306	501
4	Unemployment rate, %	min	9.3	10.3	11.6	19.3
5	Export/import ratio, %	max	0.70	0.55	0.73	0.79

Experts have chosen the following weights of these criteria values (Ginevicius *et al.* 2006): $\omega_1 = 0.28; \omega_2 = 0.19; \omega_3 = 0.15; \omega_4 = 0.18; \omega_5 = 0.20$.

We are now going to explore dependence of evaluation results using PROMETHEE I and PROMETHEE II methods on the choice of the type of the preference function $p(d)$ among the five used in practice and described above, and its parameters (Brans, Mareschal 2005; Podvezko, Podviezko 2009). The sixth *Gaussian* function was not used, as the given data does not contain standard deviation parameter σ, which also cannot be derived.

In order to choose parameters q and s for preference functions first we find out the smallest module of differences between given criteria values $\min\limits_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|$ and the largest module of differences $\max\limits_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|$ using the following algorithm. The largest module of difference could be obtained using the formula: $\max\limits_{1\le j,k\le n} \left|d_i(A_j, A_k)\right| = \max\limits_j r_{ij} - \min\limits_j r_{ij}$. For the first criterion, for example, it yields: $\max\limits_{1\le j,k\le 4} \left|d_1(A_j, A_k)\right| = 9.7 - 3.8 = 5.9$. To obtain the smallest module of difference, the data is sorted in the descending order, differences of nearby criteria values are calculated and the smallest difference is therefore taken. For example, the sorted list of values of the criterion in the first row is the following: (9.7; 7.5; 5.1; 3.8). The smallest module of differences for this criterion is equal:

$$\min_{1\le j,k\le 4 (j\ne k)} \left|d_1(A_j, A_k)\right| = \min\left|(9.7-7.5);(7.5-5.1);(5.1-3.8)\right| = \min(2.2; 2.4; 1.3) = 1.3.$$

Values of parameters q and s for preference functions are falling to the interval between the smallest and the largest modules of differences of values of criterion:

$$\min_{1\le j,k\le n} \left|d_i(A_j, A_k)\right| \le q \le s \le \max_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|. \quad (6)$$

It is clear that setting parameter q lower than just obtained the smallest value $\min\limits_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|$ and parameter s larger than the largest obtained value $\max\limits_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|$ will not make sense.

The smallest $\min\limits_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|$ and the largest $\max\limits_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|$ differences of values of criteria describing development of economies of countries (see Table 1) are shown in the Table 2.

Table 2. The smallest and the largest modules of differences between given criteria values

	Criteria	$\min\limits_{1\le j,k\le n} \left\|d_i(A_j, A_k)\right\|$	$\max\limits_{1\le j,k\le n} \left\|d_i(A_j, A_k)\right\|$
1	Annual growth of the GDP	1.3	5.9
2	Annual growth of production	1.4	9.6
3	Average annual salary in euros	8	203
4	Unemployment rate	1.0	10.0
5	Export/import ratio	0.03	0.24

To demonstrate dependence of evaluation results on the choice of preference functions and their parameters, six following examples are proposed.

The first example was already studied (Podvezko, Podviezko 2009): $p_5(d_1)$ ($q = 2$; $s = 3.5$); $p_3(d_2)$ ($s = 7$); $p_4(d_3)$ ($s = 150$); $p_2(d_4)$ ($q = 2$); $p_1(d_5)$. This means that for the first criterion the fifth preference function was used with parameters $q = 2$ and $s = 3.5$; similarly, for other criteria. We aimed to use all the five preference functions here, different for every criterion. In the second example, the first preference function was used for all criteria. It does not have q and s parameters. In the third example, the only the second preference function was used with parameters: $q_1 = 2.5$; $q_2 = 2$; $q_3 = 150$; $q_4 = 2.2$; $q_5 = 0.1$. In the fourth example the third preference function was used for all criteria with the following parameters: $s_1 = 5$; $s_2 = 8$; $s_3 = 100$; $s_4 = 10$; $s_5 = 0.1$. In the fifth example the fourth preference function was used for all the criteria with the following parameters: $q_1 = 2.5$; $s_1 = 5$; $q_2 = 2$; $s_2 = 8$; $q_3 = 130$; $s_3 = 195$; $q_4 = 2.3$; $s_4 = 10$; $q_5 = 0.06$; $s_5 = 0.15$. In the sixth example the fifth preference function was used for all the criteria with the following parameters: $q_1 = 2.5$; $s_1 = 5$; $q_2 = 2$; $s_2 = 8$; $q_3 = 130$; $s_3 = 195$; $q_4 = 2.3$; $s_4 = 10$; $q_5 = 0.06$; $s_5 = 0.15$.

In different fourth and fifth preference functions used in fifth and sixth examples, we chose the same parameters q and s.

Now we find out dominance relation $\pi(A_j, A_k)$ between all pairs of alternatives: preference, indifference and incomparability by using the formula (2). Then assessment of outranking flows F^+ and F^-, respectively positive and negative is made. Results are given in the Table 3.

It is clearly observed that outranking flows used in both PROMETHEE I and PROMETHEE II methods F_j^+, F_j^- and F_j considerably differ between themselves. Ranks are not always matching as well, when different preference functions are used (the third, the fourth and the fifth examples). Note that in the fifth and the sixth examples exposed in Table 3 two different preference functions were used (the fourth and the fifth) with the same parameters q and s, and this yielded different outcome.

Observe dependence of the result of evaluation on choice of parameters as well as on choice of the type of preference function. It is interesting to look simultaneously to influences of both the fourth and the fifth preference functions, which depend on two parameters q and s. The two functions differ in the interval $[q, s]$, where the fifth function uniformly increases in accordance with the expression $\frac{d-q}{s-q}$, as the difference of criteria values d increase, while the fourth function assigns the same average value of 0.5 in the interval. First, for every i-th criterion let us choose the largest possible interval $[q,s] = \left[\min_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|, \max_{1\le j,k\le n} \left|d_i(A_j, A_k)\right|\right]$ (see Table 2) and then diminish the interval, at each grade extinguishing worst or best alternative at the time. We obtain the following outcome (Table 4).

Our carried out computations display the fact that evaluation results may well differ upon the choice of preference functions as well as on their parameters q and s. Outranking flow values F_j^+, F_j^- and mostly F_j can considerably differ. Yielded evaluation ranks of countries can also differ. In spite of the fact that Lithuania outranks other countries by economic criteria of 2003, ranks of other countries depend on choice of preference function and chosen values of parameters q and s.

Table 3. Evaluations with different preference functions

Examples	Evaluation outcome	Estonia	Latvia	Lithuania	Poland
1. All preference functions are different	F_j^+	0.838	0.535	1.728	1.027
	F_j^-	1.001	1.193	0.605	1.325
	F_j	–0.163	–0.658	1.123	–0.293
	PROMETHEE I (ranks)	2	–	1	–
	PROMETHEE II (ranks)	2	4	1	3
2. The first preference function for all criteria	F_j^+	1.70	0.92	2.14	1.24
	F_j^-	1.30	2.08	0.86	1.76
	F_j	0.40	–1.16	1.28	–0.52
	PROMETHEE I (ranks)	2	–	1	3
	PROMETHEE II (ranks)	2	4	1	3
3. The second preference function for all criteria: $q_1 = 2.5$; $q_2 = 2$; $q_3 = 150$; $q_4 = 2.2$; $q_5 = 0.1$	F_j^+	0.75	0.46	1.51	0.50
	F_j^-	0.47	1.13	0.33	1.29
	F_j	0.28	–0.67	1.18	–0.79
	PROMETHEE I (ranks)	2	–	1	–
	PROMETHEE II (ranks)	2	3	1	4
4. The third preference function for all criteria: $s_1 = 5$; $s_2 = 8$; $s_3 = 100$; $s_4 = 10$; $s_5 = 0.1$	F_j^+	0.924	0.527	1.594	0.952
	F_j^-	0.888	1.367	0.485	1.257
	F_j	0.036	–0.840	1.109	–0.305
	PROMETHEE I (ranks)	2–3	–	1	2–3
	PROMETHEE II (ranks)	2	4	1	3
5. The fourth preference function for all criteria: $q_1 = 2.5$; $s_1 = 5$; $q_2 = 2$; $s_2 = 8$; $q_3 = 130$; $s_3 = 195$; $q_4 = 2.3$; $s_4 = 10$; $q_5 = 0.06$; $s_5 = 0.15$	F_j^+	0.450	0.230	1.090	0.625
	F_j^-	0.335	1.010	0.265	0.785
	F_j	0.115	–0.780	0.825	–0.160
	PROMETHEE I (ranks)	2–3	–	1	2–3
	PROMETHEE II (ranks)	2	4	1	3
6. The fifth preference function for all criteria: $q_1 = 2.5$; $s_1 = 5$; $q_2 = 2$; $s_2 = 8$; $q_3 = 130$; $s_3 = 195$; $q_4 = 2.3$; $s_4 = 10$; $q_5 = 0.06$; $s_5 = 0.15$	F_j^+	0.426	0.291	1.348	0.567
	F_j^-	0.438	0.986	0.150	1.058
	F_j	–0.012	–0.695	1.198	–0.491
	PROMETHEE I (ranks)	2	–	1	–
	PROMETHEE II (ranks)	2	4	1	3

Table 4. Influence of choice of parameters to the evaluation outcome

Interval of parameters $[q_i, s_i]$	Evaluation outcome	The fourth function				The fifth function			
		Estonia	Latvia	Lithuania	Poland	Estonia	Latvia	Lithuania	Poland
1) [1.3;5.9] 2) [1.4;9.6] 3) [8;203] 4) [1;10] 5) [0.03;0.24]	F_j^+	0.620	0.460	0.995	0.720	0.549	0.379	1.262	0.640
	F_j^-	0.650	0.975	0.430	0.740	0.487	1.003	0.294	1.046
	F_j	–0.030	–0.515	0.565	–0.02	0.062	–0.624	0.968	–0.406
	PROMETHEE I (ranks)	2–3	–	1	2–3	3	–	1	–
	PROMETHEE II (ranks)	3	4	1	2	2	4	1	3
1) [2.2;4.6] 2) [1.9;7.7] 3) [71;195] 4) [1.3;9] 5) [0.06;0.18]	F_j^+	0.615	0.460	1.085	0.625	0.537	0.378	1.434	0.550
	F_j^-	0.475	0.910	0.430	0.970	0.497	1.010	0.237	1.155
	F_j	0.140	–0.450	0.655	–0.345	0.040	–0.631	1.196	–0.605
	PROMETHEE I (ranks)	2	–	1	–	2	–	1	–
	PROMETHEE II (ranks)	2	4	1	3	2	4	1	3
1) [2.4;3.7] 2) [3.3;6.3] 3) [124;132] 4) [2.3;7.7] 5) [0.09;0.15]	F_j^+	0.540	0.320	1.325	0.600	0.530	0.460	1.510	0.500
	F_j^-	0.475	1.010	0.240	1.060	0.470	1.090	0.150	1.290
	F_j	0.065	–0.690	1.085	–0.460	0.060	–0.630	1.360	–0.790
	PROMETHEE I (ranks)	2	–	1	–	2	–	1	–
	PROMETHEE II (ranks)	2	4	1	3	2	3	1	4

5. Conclusions

PROMETHEE methods fall to the range of complex quantitative multi-criteria methods. They account values of criteria (and their weights) indirectly over so-called preference functions. Computations of different examples reveal the fact that evaluation outcome depends on both choice of preference function and its parameters. What is the most important, choices cannot be made carelessly. Unlike other popular multi-criteria methods, active participation of decision-makers or qualified specialists is compulsory as they recommend types of preference functions for every criterion, set the largest and the lowers boundaries for all criteria parameters as well as other parameters. New tools were proposed in this paper, new types of preference functions, with intention of widening the range of choice for decision-makers and evaluation experts. An algorithm yielding the largest and the lowest boundaries for parameters of preference functions thus helping to make a choice of these parameters is also presented.

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DAUGIAKRITERINIŲ VERTINIMŲ REZULTATŲ PRIKLAUSOMYBĖ NUO PRIORITETŲ FUNKCIJŲ IR JŲ PARAMETRŲ PASIRINKIMO

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Santrauka

Pastaruoju metu socialinių ir ekonominių sudėtingų reiškinių kiekybiniam lyginimui plačiai taikomi daugiakriteriniai metodai. PROMETHEE metodai skiriasi nuo kitų daugiakriterinių metodų savo ypatumu ir gilesne logika. Metodo pagrindą sudaro vadinamosios prioritetų funkcijos. Jų tipai pasirenkami ir jų parametrai nustatomi aktyviai dalyvaujant priimantiems sprendimą asmenims. Tai yra PROMETHEE metodų privalumas bei ypatumas. Darbe parodyta vertinimų rezultatų priklausomybė nuo prioritetų funkcijų ir jų parametrų reikšmių pasirinkimo. Kartu su autorių anksčiau nagrinėtu PROMETHEE I metodu šiame darbe aprašytas PROMETHEE II metodas, pateiktas jo taikymo pavyzdys ranguojant alternatyvas.

Reikšminiai žodžiai: prioritetų funkcijos, prioritetų funkcijų parametrai, daugiakriteriniai vertinimai, PROMETHEE metodai.

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