JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
2010
16(4): 451–461

PROBABILITY-BASED DESIGN OF SPUN CONCRETE BEAM-COLUMNS

Antanas Kudzys[1], Romualdas Kliukas[2]
[1]*Kaunas University of Technology, Institute of Architecture and Construction, Tunelio g. 60, 44405 Kaunas, Lithuania*
[2]*Vilnius Gediminas Technical University, Saulėtekio al. 11, 10223 Vilnius, Lithuania*
E-mails: [1]antanas.kudzys@gmail.com; [2]pirmininkas@vgtu.lt
Received 16 March 2010; accepted 04 May 2010

Abstract. The attention of engineers is turned to the application of precast spun concrete columns reinforced by high-strength steel bars for office and administrative buildings. The paper discusses a possibility of using the reliability index approach to designing beam-columns of building frames and analyses resistance criteria for beam-columns of annular cross sections as compression members with bending moments and bending members with compressive forces. First and second order effects of beam-columns are considered. The article also presents time-dependent resistance safety margin and its stationary equivalent and investigates the unsophisticated applied models for probability-based design of frame beam-columns. The design of a beam-column of the braced frames is illustrated providing a numerical example.

Keywords: spun concrete, beam-columns, frames, second order effects, resistance safety margin, reliability index design.

1. Introduction

The economically and structurally rational precast spun concrete columns reinforced by longitudinal high-strength steel bars uniformly distributed throughout their annular cross sections may be treated as sustained beam-columns of office and administrative buildings (Kudzys and Kliukas 2009). High-strength reinforcing steel bars increase the ductility and capacity of eccentrically loaded columns due to the redistribution of ultimate compressive stresses of steel and concrete components of beam-columns (MacGregor 1988; Kudzys *et al.* 1993; Kliukas *et al*. 2010; Židonis 2009; Juocevičius and Vaidogas 2010).

The Standards EN 1990 (2002) in Europe and ASCE/SEI 7-05 (2006) in the USA require that the load carrying structures of buildings shall be designed with the appropriate degrees of reliability. However, these standards are based on limit state concepts and respectively on the methods of partial safety factors design and load and resistance factors design. Therefore, some contradictions in reliability approaches presented in these Standards and the International Standard ISO 2394 (1998) on the reliability for structures exist. Practically, the reliability degree of load-carrying structures designed by various native semi-probabilistic and full-probabilistic limit state concepts can be markedly different in their values. This difference depends on the conditionality of mechanical and statistical uncertainties evaluated and integrated in design models.

The quantitative reliability indices of particular members (sections) and structural members (columns, beams) may be objectively defined and predicted only by full-probabilistic approaches and models. However, the recommendations and directions devoted to the issues of design features of probability-based approaches presented in the design codes of reinforced concrete structures Eurocode (EN 1992-1 2004) and ACI Committee 318-05 (2005) are not fully formulated. Therefore, any possibility of engineering and wish for an objective prediction of the probabilistic parameters of building structures, including beam-columns, are rather hardly interpreted and used in design practice (Vaidogas and Juocevičius 2008; Jankovski and Atkočiūnas 2008).

In spite of the fairly developed concepts of probabilistic reliability design, it is difficult to apply probability-based approaches in structural safety predictions both of complicated and not complicated members and their systems. These approaches may be acceptable to building engineers only under the indispensable condition that the probabilistic performances of members may be considered in design practice using unsophisticated and easy perceptible manners.

The main task of this paper is to present new methodological formats on the probability-based reliability predictions of the beam-columns of reinforced concrete frames subjected to recurrent single and joint extreme service and climate actions. The paper considers the possibility of using reliability index design approaches based on the transformed conditional intersection in engineering practice and failure probabilities of members and the stationary processes of their resistance safety margins using equivalent recurrent extreme service actions instead of their sustained and extraordinary components.

ISSN 1392–3730 print / ISSN 1822–3605 online
doi:10.3846/jcem.2010.51

2. Resisting Compressive Forces and Bending Moments

2.1. Compression with a Bending Moment

Beam-columns are represented in design state taking into account their particular members (normal sections). Therefore, the survival probabilities of beam-columns may be objectively assessed and predicted only knowing mechanical and statistical parameters of their normal sections.

Modelling the stress-strain state of spun concrete beam-columns must assess the structural features of annular cross sections reinforced by high-strength steel bars. According to the interaction diagram $N_R - M_R$ of eccentrically compressed spun concrete members, segments AB and B–C characterize their fully and partially compressed cross sections (Fig. 1).

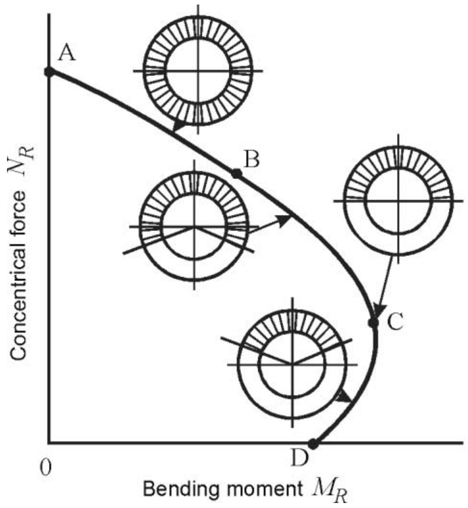

Fig. 1. Interaction $N_R - M_R$ diagram of eccentrically loaded annular cross sections

When the action effects of building structures are provoked only by gravity loads, beam-columns are under compression with a small bending moment. Modelling strain and stress distributions in concrete and high-strength steel bars and the bearing capacity of eccentrically loaded sections may be based on a plain cross-section hypothesis and bi-linear concrete strain-stress relation (Fig. 2).

When eccentricity ratio $e/r_s \le 1.0$, the ultimate internal resisting compressive force of annular cross sections may be expressed as

$$R_N = N_R = \left(k_c f_{cc} A_c + k_s \sigma_{sc} A_s\right) r_s / \left(e + r_s\right), \quad (1)$$

where the response factors characterizing contributions of concrete and steel components to the ultimate resistance of beam-columns are defined as

$$k_c = 1 - \left(0.3e/r_s\right)/\left(1 + 10\rho\right), \quad (2)$$

$$k_s = 1 - 0.34 e/r_s, \quad (3)$$

$$f_{cc} = \alpha_{cc}\alpha_2 f_c = \left(1 - 0.1 N_{EP}/N_E\right)\left(0.85 - 1.7\rho\right) f_c, \quad (4)$$

is the compressive strength of spun concrete in beam-columns the cylinder strength of which is f_c; N_E and N_{EP} are the applied total force and its quasi-permanent component;

$$\sigma_{sc} = 452\left(1.36 + 4\rho\right) \text{ MPa}, \quad (5)$$

is ultimate compressive stress in reinforcing bars; $\rho = A_s/A_c$ is reinforcement ration; A_c, A_s and r_s are geometrical parameters taken from Fig. 2.

The mean and variance of response R_N by Eq. (1) are

$$R_{Nm} = \left(k_{cm} f_{ccm} A_{cm} + k_{sm}\sigma_{scm} A_s\right) r_s / \left(e_m + r_s\right), \quad (6)$$

$$\boldsymbol{\sigma}^2 R_N = \left(\frac{\partial R_N}{\partial f_{cc}}\right)_m^2 \boldsymbol{\sigma}^2 f_{cc} + \left(\frac{\partial R_N}{\partial A_c}\right)_m^2 \boldsymbol{\sigma}^2 A_c + \left(\frac{\partial R_N}{\partial \sigma_{sc}}\right)_m^2 \boldsymbol{\sigma}^2 \sigma_{sc} + \left(\frac{\partial R_N}{\partial e}\right)_m^2 \boldsymbol{\sigma}^2 e + \left(\frac{\partial R_N}{\partial \rho}\right)_m^2 \boldsymbol{\sigma}^2 \rho + 2\left[\left(\frac{\partial R_N}{\partial A_c}\right)_m \left(\frac{\partial R_N}{\partial \rho}\right)_m \boldsymbol{\sigma} A_c \times \boldsymbol{\sigma}\rho + \left(\frac{\partial R_N}{\partial \sigma_{sc}}\right)_m \left(\frac{\partial R_N}{\partial \rho}\right)_m \boldsymbol{\sigma}\sigma_{sc} \times \boldsymbol{\sigma}\rho\right], \quad (7)$$

where k_{cm}, k_{sm}, f_{ccm} and σ_{scm} are taken from Eqs. (2)–(5);

$$\boldsymbol{\delta} f_c = 0.088 + 0.00003\left(70 - f_{ck}\right)^2, \quad (8)$$

$$\boldsymbol{\delta} f_{cc} = \left(\boldsymbol{\delta}^2 f_c + 0.08^2\right)^{1/2}, \; \boldsymbol{\sigma}^2 f_{cc} = \left(\boldsymbol{\delta} f_{cc} \times f_{ccm}\right)^2, \quad (9)$$

$$\boldsymbol{\delta} A \approx \boldsymbol{\delta} I = \left(1.2 - r_2\right)/\left[150\left(r_2 - r_1\right)\right], \; \boldsymbol{\sigma}^2 A_c = \left(\boldsymbol{\delta} A \times A_{cm}\right)^2, \quad (10)$$

$$\boldsymbol{\sigma}^2 \sigma_{sc} = \left(\boldsymbol{\delta}\sigma_{sc} \times \sigma_{scm}\right)^2 = \left(0.105 \times \sigma_{scm}\right)^2, \quad (11)$$

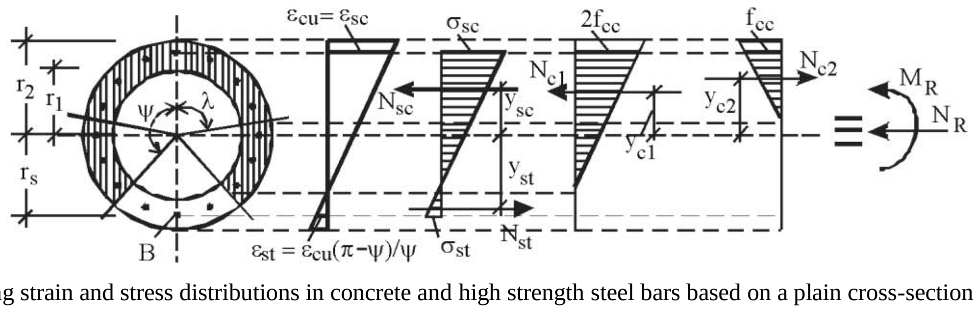

Fig. 2. Modelling strain and stress distributions in concrete and high strength steel bars based on a plain cross-section hypothesis

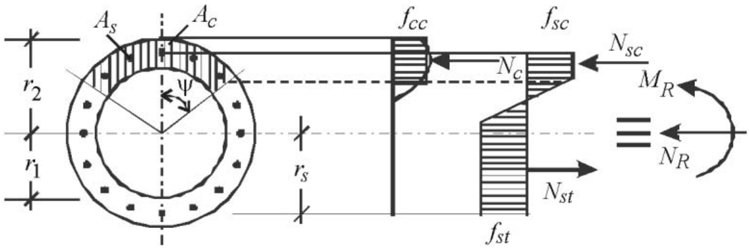

Fig. 3. Modelling stresses in concrete and steel bars based on their simplified plastic distributions

e_m by Eq. (35), $\boldsymbol{\sigma}^2 e$ by Eq. (36),

$$\rho_m = A_s / A_{cm}, \ \boldsymbol{\sigma}^2 \rho = \left(A_s / A_{cm}^2\right)^2 \boldsymbol{\sigma}^2 A_c . \qquad (12)$$

It must be noted that the coefficient of variation in spun concrete strength in thin-walled members increases and is equal to $\boldsymbol{\delta} f_{cc} = \left(\boldsymbol{\delta}^2 f_c + \boldsymbol{\delta}^2 f_{cr}\right)^{1/2}$ where components $\boldsymbol{\delta} f_c$ and $\boldsymbol{\delta} f_{cr}$ indicate the effects of concrete mix quality and reinforcement presence on its value.

2.2. Bending with Compressive Forces

According to Vadlūga (1979), the ultimate internal resisting bending moment M_R of the beam-columns of annular cross sections (Fig. 3) may be plastic and expressed as

$$R_M = M_R = 1.2 r_s \left(A_s f_{st} + N_E\right) \times \left[1 - \frac{A_{st} f_{st} + N_E}{A_c f_{cc} + A_s \left(f_{st} + f_{sc}\right)}\right]. \qquad (13)$$

For design practice, the mean of this resistance may be written in the form

$$R_{Mm} = T_{2m} T_{3m} / T_{1m}, \qquad (14)$$

where

$$T_{1m} = A_{cm} f_{ccm} + A_s \left(f_{stm} + f_{scm}\right), \qquad (15)$$

$$T_{2m} = 1.2 r_s \left(A_s f_{stm} + N_{Em}\right), \qquad (16)$$

$$T_{3m} = A_{cm} f_{ccm} + A_s f_{scm} - N_{Em}, \qquad (17)$$

when f_{ccm} is defined from Eq. (4); $f_{stm} = 500$ MPa, $f_{scm} = 600$ MPa; N_{Em} is joint compressive force provoked by service and climate actions.

The variance of resistance by Eq. (13) is expressed as

$$\boldsymbol{\sigma}^2 R_M = \left[\frac{T_{2m}\left(T_{1m} - T_{3m}\right)}{T_{1m}^2}\right]^2 \times \left(A_{cm}^2 \boldsymbol{\sigma}^2 f_{cc} + f_{ccm}^2 \boldsymbol{\sigma}^2 A_c + A_s^2 \boldsymbol{\sigma}^2 f_{sc}\right) + \left[\frac{A_s T_{3m}\left(1.2 r_s T_{1m} - T_{2m}\right)}{T_{1m}^2}\right]^2 \boldsymbol{\sigma}^2 f_{st} + \left[\frac{1.2 r_s T_{3m} - T_{2m}}{T_{1m}}\right]^2 \boldsymbol{\sigma}^2 N_E, \qquad (18)$$

where k_{cm}, k_{sm}, f_{ccm} and σ_{scm} are taken from Eqs. (2)–(5); $\boldsymbol{\sigma}^2 f_{cc}$ by Eq. (9); $\boldsymbol{\sigma}^2 A_c$ by Eq. (10);

$$\boldsymbol{\sigma}^2 f_{st} = \left(\boldsymbol{\delta} f_s \times f_{stm}\right)^2, \ \boldsymbol{\sigma}^2 f_{sc} = \left(\boldsymbol{\delta} f_s \times f_{scm}\right)^2 . \qquad (19)$$

where $\boldsymbol{\delta} f_s = \left(\boldsymbol{\delta}^2 f_{s1} + \boldsymbol{\delta}^2 f_{s2}\right)^{1/2}$ when components $\boldsymbol{\delta} f_{s1} = 0.09$ and $\boldsymbol{\delta} f_{s2} = 0.12$ define statistical deviations and uncertainties of right-angled epures of steel bar stresses.

3. First and Second Order Effects

The first order mean value of the eccentricity of the applied total compressive force N_{Em} is equal to

$$e_{om} = M_{OEm} / N_{Em} \qquad (20)$$

but not less as $r_2/15$ and 20 mm (EN 1992-1 2004).

The second order eccentricity of this force may be expressed as a magnification of the first order eccentricity resulting from linear-elastic analysis with redistribution in which the internal moments are modified with external actions and without a more explicit calculation of rotation capacity (EN 1990 2002; Kargaudas and Adamukaitis 2010).

According to EN 1992-1 (2004), the additional action effects of reinforced concrete beam-columns may be estimated by the flexural stiffness method. This stiffness of slender compression elements with constant cross sections may be represented as

$$(EI) = K_c E_c I_c + E_s I_s, \qquad (21)$$

where

$$K_c = 0.25 / \left(1 + \varnothing M_{OG} / M_{OE}\right), \qquad (22)$$

is the factor for effects of cracking and creep deformations on the overall behaviour of members; M_{OG} and M_{OE} are bending moments caused by permanent and total actions; $\varnothing = 1.2$–2.0 is the basic creep coefficient of concrete the value of which depends on its strength class and dimensions of a cross section of members; E_c, E_s and I_c, I_s are the tangent module of elasticity and the second moments of the areas of concrete and steel bars.

The mean and variance of flexural stiffness by Eq. (21) may be expressed as

$$(EI)_m = K_{cm} E_{cm} I_{cm} + E_s I_s, \qquad (23)$$

$$\boldsymbol{\sigma}^2(EI) = (K_{cm}E_{cm})^2\boldsymbol{\sigma}^2 I_c + (K_{cm}I_{cm})^2\boldsymbol{\sigma}^2 E_c + (E_{cm}I_{cm})^2\boldsymbol{\sigma}^2 K_c, \quad (24)$$

where

$$K_{cm} = 0.25/(1+\varnothing M_{OGm}/M_{OEm}), \quad (25)$$

$$\boldsymbol{\sigma}^2 K_c = \left[(0.25\varnothing M_{OGm})/(M_{OEm}+\varnothing M_{OGm})^2\right]^2 \times (\boldsymbol{\sigma}^2 N_G + \boldsymbol{\sigma}^2 N_E), \quad (26)$$

$$E_{cm} = 20(0.1 f_{cm})^{0.3}, \quad (27)$$

$$\boldsymbol{\sigma}^2 E_c = (0.15E_{cm})^2, \quad (28)$$

$$I_{cm} = \pi\left(r_2^4 - r_{1m}^4\right)/4, \quad (29)$$

$$\boldsymbol{\sigma}^2 I_c = (1.20 - r_2)/[150(r_2 - r_{1m})], \quad (30)$$

$$I_s = \pi\left(r_{s2}^4 - r_{s1}^2\right)/4. \quad (31)$$

Buckling resistance $N_B = \pi^2(EI)/l_o^2$ can be used as a mechanical parameter in the second order analysis of members the mean and variance of which are

$$N_{Bm} = \pi(EI)_m / l_{om}^2, \quad (32)$$

$$\boldsymbol{\sigma}^2 N_B = \pi^2\boldsymbol{\sigma}^2(EI)/l_{om}^2 + \left[2\pi^2(EI)_m/l_o^3\right]^2\boldsymbol{\sigma}^2 l_o, \quad (33)$$

where $(EI)_m$ and $\boldsymbol{\sigma}^2(EI)$ are defined by Eqs. (23) and (24); $l_{om} = 0.75h_s$ and $l_{om} = h_s$ are the effective length of the columns of buildings with in situ and precast floor beams; h_s is a storey height of a building.

For beam-columns without transverse loads, the second order eccentricity may be defined by the equation

$$e = e_o\left[1 + \frac{\pi^2/c}{(N_B/N_E - 1)}\right], \quad (34)$$

where parameter $c = 8$–12 depends on the distribution of moments and π^2/c is normally a reasonable simplification. Therefore, the mean and variance of this eccentricity are

$$e_m = e_{om}\left(\frac{1}{1 - N_{Em}/N_{Bm}}\right), \quad (35)$$

$$\boldsymbol{\sigma}^2 e = \left[\frac{N_{Em}e_{om}}{(N_{Bm}-N_{Em})^2}\right]^2\boldsymbol{\sigma}^2 N_B + \left[\frac{N_{Bm}e_{om}}{(N_{Bm}-N_{Em})^2}\right]^2\boldsymbol{\sigma}^2 N_E, \quad (36)$$

where N_{Bm}, $\boldsymbol{\sigma}^2 N_B$ and N_{Em}, $\boldsymbol{\sigma}^2 N_E$ are the statistics of buckling resistance and the total compressive force of a beam-column.

Design limit states of beam-columns include their loss of equilibrium and large deformations leading them to the second order effects. The model of the structural response of beam-columns reinforced by high-strength steel bars may be based on linear-elastic (Fig. 2) and non-linear (plastic) (Fig. 3) resistances.

4. Resistance Safety Margins

According to probability-based approaches, the resistance safety margin of the particular members of the beam-columns of buildings is a non-stationary resistance performance process presented as:

$$Z(t) = g[\mathbf{X}(t), \boldsymbol{\theta}] = \theta_R R - \theta_G E_G - \theta_{Q_s}E_{Q_s}(t) - \theta_{Q_e}E_{Q_e}(t) - \theta_{cl}E_{cl}, \quad (37)$$

where $\mathbf{X}(t)$ and $\boldsymbol{\theta}$ are the random vectors of basic and additional variables representing mechanical parameters and their model uncertainties. These parameters include resistance R_N by Eq. 1 or R_M by Eq. (13) and action effects E_G, E_{Q_s}, E_{Q_e}, E_{cl} caused by permanent, G, floor sustained, Q_s, and extraordinary, Q_e, live loads and climate actions (snow, S, wind, W, loads) (Fig. 4).

Live floor loads vary in time and space in a random manner (JCSS 2000). The sustained part of these loads in civil and engineering buildings contains the weight of furniture and heavy equipments respectively. An intermittent extraordinary load represents all kinds of live floor loads which are not covered by the sustained load. It is caused by the crowded rooms during special events and by mobile equipment during processes in civil and engineering buildings respectively.

The probability distribution of resistance, R, and permanent action effect, E_G, of reinforced concrete members is close to normal distribution (Ramsay *et al*. 1979; Вадлуга 1979; Ellingwood 1981; ISO 2394 1988; EN 1990 2002; EN 1992-1 2004; JCSS 2000). According to these international standards, Gaussian or lognormal distributions may be assumed for sustained actions and Weibull, gamma or exponential distributions may be used for extraordinary actions representing non-stationary processes.

Analogically to traditional standard approaches of design codes, the probability-based design needs simplifications of safety margin processes of particular members.

The annular extreme sum of sustained and extraordinary live action effects may be modeled as a stationary rectangular wave renewal process described by Type 1 (Gumbel) distribution (Rosowsky and Ellingwood 1992). The mean and coefficient of variation as well as variance in this process may be defined as

$$E_{Qm} = E_{Q_s m}(t_i) + E_{Q_e m}(t_i) = E_{Q_s m}(t_j) + E_{Q_e m}(t_j) = \frac{E_{Q_s k}}{1 + k_{0.95}^{Q_s}\boldsymbol{\delta}E_{Q_s}} + \frac{E_{Q_e k}}{1 + k_{0.95}^{Q_e}\boldsymbol{\delta}E_{Q_e}}, \quad (38)$$

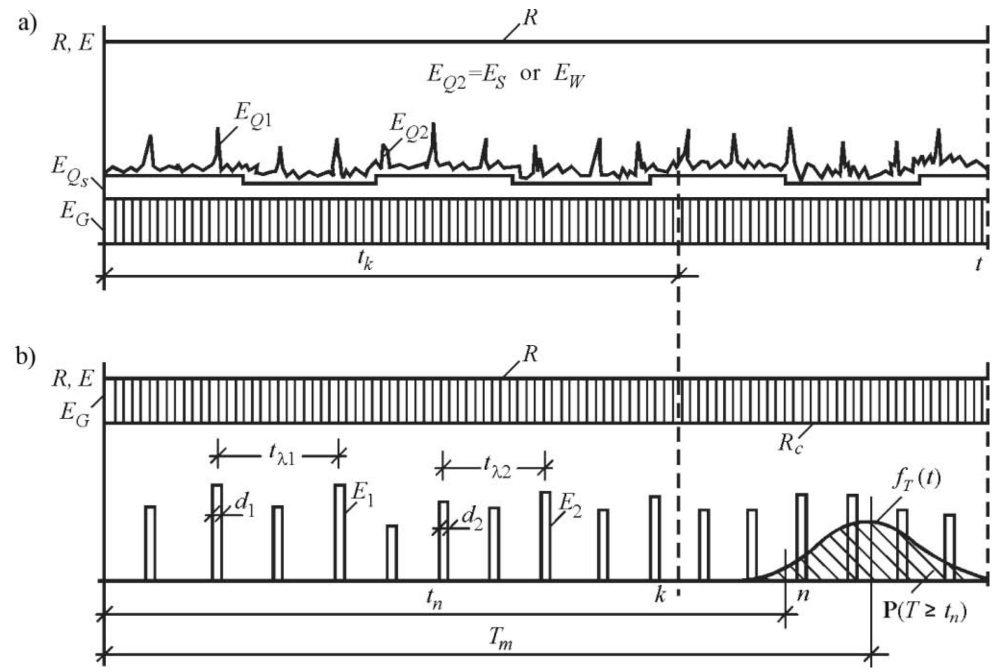

Fig. 4. Actual (a) and applied (b) models for a time-dependent safety analysis of particular members

$$\boldsymbol{\delta}E_Q = \frac{\boldsymbol{\sigma}E_Q}{E_{Qm}} = \frac{\left(\boldsymbol{\sigma}^2 E_{Q_s} + \boldsymbol{\sigma}^2 E_{Q_e}\right)^{1/2}}{E_{Qm}} =$$

$$\frac{\left[\left(\boldsymbol{\delta}E_{Q_s} \times E_{Q_s m}\right)^2 + \left(\boldsymbol{\delta}E_{Q_e} \times E_{Q_e m}\right)^2\right]^{1/2}}{E_{Qm}}, \qquad (39)$$

$$\boldsymbol{\sigma}^2 E_Q = \left(\boldsymbol{\delta}E_Q \times E_{Qm}\right)^2, \qquad (40)$$

where $E_{Q_s k}$ and $E_{Q_e k}$ are the characteristic (nominal) values of live load components; $k_{0.95}^{Q_s}$ and $k_{0.95}^{Q_e}$ are the characteristic fractile of their probability distributions; $\boldsymbol{\delta}E_{Q_s}$ and $\boldsymbol{\delta}E_{Q_e}$ are the coefficients of variation in these components.

When $E_{Q_s m} = 18.55$ kN, $\boldsymbol{\delta}E_{Q_s} = 0.9$, $E_{Q_e m} = 12.0$ kN, $\boldsymbol{\delta}E_{Q_e} = 0.5$, $E_{Qm} = 18.55 + 12.0 = 30.55$ kN and $E_{Qk} = 65$ kN, then according to Eq. (39), the coefficient of variation in the stationary extreme floor action, E_Q, is equal to

$$\boldsymbol{\delta}E_Q = \left[(18.55 \times 0.9)^2 + (12.0 \times 0.5)^2\right]^{1/2} / 30.55 = 0.58.$$

Values $E_{Qm} = 0.47 E_{Qk}$ and $\boldsymbol{\delta}E_Q = 0.58$ were suggested by Rosowsky and Ellingwood (1992).

Gumbel cumulative distribution is quite appropriate for the annual extreme snow and wind loads (Ellingwood 1981; JCSS 2000; Vrouwenvelder 2002). The mean values, coefficients of variation and variances in these load effects may be expressed as

$$E_{sm} = E_{sk} / \left(1 + k_{0.98}^{s} \boldsymbol{\delta}E_s\right), \ \boldsymbol{\delta}E_s = 0.3 - 0.7, \qquad (41)$$

$$E_{wm} = E_{wk} / \left(1 + k_{0.98}^{w} \boldsymbol{\delta}E_w\right), \ \boldsymbol{\delta}E_w = 0.2 - 0.5, \qquad (42)$$

$$\boldsymbol{\sigma}^2 E_s = \left(\boldsymbol{\delta}E_s \times E_{sm}\right)^2, \qquad (43)$$

$$\boldsymbol{\sigma}^2 E_w = \left(\boldsymbol{\delta}E_w \times E_{wm}\right)^2. \qquad (44)$$

Besides, a coincidence of annual extreme wind and snow loads is impossible. However, the recurrence number of joint extreme floor and climate action effects during the working life t_n of beam-columns may be defined as

$$n_{Q,cl} = t_n \left(d_Q + d_{cl}\right) \lambda_Q \lambda_{cl}, \qquad (45)$$

where $d_Q = 1$–3 days, $d_{cl} = d_s = 14$–28 days and $d_{cl} = d_w = 8$–12 hours are the durations of extreme action effects; $\lambda_Q = 1$ and $\lambda_{cl} = 1$ are their renewal rates.

Therefore, $n_{Qs} = 2.06$–4.25 and $n_{Qw} = 0.18$–0.48.

Thus, extreme events arise from extraordinary floor and climate conditions that usually are not considered explicitly in the limit state design of buildings. The mean and variance of stationary bivariate annual extreme action effects are

$$E_{cm} = E_{Qm} + E_{Sm} \text{ or } E_{cm} = E_{Qm} + E_{Wm}, \qquad (46)$$

$$\boldsymbol{\sigma}^2 E_c = \boldsymbol{\sigma}^2 E_Q + \boldsymbol{\sigma}^2 E_S \text{ or } \boldsymbol{\sigma}^2 E_c = \boldsymbol{\sigma}^2 E_Q + \boldsymbol{\sigma}^2 E_W, \qquad (47)$$

where E_{Qm}, $\boldsymbol{\sigma}^2 E_Q$, E_{Sm}, $\boldsymbol{\sigma}^2 E_S$, E_{Wm} and $\boldsymbol{\sigma}^2 E_W$ are defined by Eqs. (38), (40), (41), (43), (42) and (44).

The combination of short episodic annual extreme action effects of beam-columns belongs to exceptional events when their magnitude can be considerable but the probability of the occurrence of which is small and related to the working time of load-carrying structures. These action effects help us with rewriting the non-stationary safety margin by Eq. (37) in the stationary form

$$Z = \theta_R R - \theta_G E_G - \theta_Q E_Q - \theta_{cl} E_{cl} = R_C - E_C , \quad (48)$$

where

$$R_C = \theta_R R - \theta_G E_G , \quad (49)$$

is the conventional resistance of beam-columns,

$$E_C = \theta_Q E_Q + \theta_{cl} E_{cl} \text{ or } E_C = \theta_Q E_Q \text{ or } E_C = \theta_{cl} E_{cl} , \quad (50)$$

is conventional extreme action effects described by Gumbel distribution law.

In design practice of beam-columns, additional variables θ_i may be represented by their probability density functions or simply as their means and standard deviations, θ_{im} and $\boldsymbol{\sigma}\theta_i$ (see Section 6).

The means and variances of conventional parameters R_c and E_c of beam-columns may be defined as

$$R_{cm} = \theta_{Rm} R_m - \theta_{Gm} E_{Gm} , \quad (51)$$

$$\boldsymbol{\sigma}^2 R_c = \theta_{Rm}^2 \boldsymbol{\sigma}^2 R + R_m^2 \boldsymbol{\sigma}^2 \theta_R + \theta_{Gm} \boldsymbol{\sigma}^2 E_G + E_{Gm}^2 \boldsymbol{\sigma}^2 \theta_G , \quad (52)$$

$$E_{Cm} = \theta_{Qm} E_{Qm} + \theta_{cl,m} E_{cl,m} , \quad (53)$$

$$\boldsymbol{\sigma}^2 E_c = \theta_{Qm}^2 \boldsymbol{\sigma}^2 E_Q + E_{Qm}^2 \boldsymbol{\sigma}^2 \theta_Q + \theta_{cl,m}^2 \boldsymbol{\sigma}^2 E_{cl} + E_{cl,m}^2 \boldsymbol{\sigma}^2 \theta_{cl} , \quad (54)$$

where R_m by Eq. (6) or (14); $E_{Gm} = N_{Gk}$ or $E_{Gm} = M_{Gk} = N_{Gk} e$; E_{Qm} by Eq. (38), $\boldsymbol{\sigma}^2 E_Q$ by Eq. (40); $E_{cl,m}$ by Eq. (41) or (42), $\boldsymbol{\sigma}^2 E_{cl}$ by Eq. (43) or (44).

5. Instantaneous and Long Term Survival Probabilities

When extreme action effects are caused by two stochastically independent variable actions, a failure of beam-columns may occur not only in the case of their coincidence but also when the value of one out of two effects is extreme. Therefore, three stochastically dependent resistance safety margins should be considered as follows:

$$Z_{k_Q} = R_C - E_{Qk_Q} , \; k_Q = 1, \; 2, \; \ldots, \; n_Q , \quad (55)$$

$$Z_{k_{cl}} = R_C - E_{cl,k_{cl}} , \; k_{cl} = 1, \; 2, \; \ldots, \; n_{cl} , \quad (56)$$

$$Z_{k_{Q,cl}} = R_C - E_{Q,cl,k_{Q,cl}} , \; k_{Q,cl} = 1, \; 2, \; \ldots, \; n_{Q,cl} . \quad (57)$$

The instantaneous survival probability of beam-columns may be represented by the convolution integral as

$$\mathbf{P}(S_k) = \mathbf{P}(R_C > E_C) = \int_0^{\infty} f_{R_C}(x) F_{E_C}(x) dx , \quad (58)$$

where $f_{R_c}(x)$ is the density function of their conventional resistance.

$$R_{CN} = \theta_{R_N} R_N - \theta_{N_G} N_G \text{ or } R_{CM} = \theta_{R_M} R_M - \theta_{M_G} M_G .$$

$$F_{E_C}(x) = \exp\left[-\exp\left(\frac{E_{Cm} - x}{0.7794 \boldsymbol{\sigma} E_C} - 0.5772 \right) \right] , \quad (59)$$

is the cumulative distribution function for their conventional action effect $N_C = \theta_{N_Q} N_Q + \theta_{N_{cl}} N_{cl}$ or $M_C = \theta_{M_Q} M_Q + \theta_{M_{cl}} M_{cl}$.

The stochastically dependent instantaneous survival probabilities by Eq. (58) of beam-columns exposed to the interrupted extreme action effects form series systems with n_Q, n_{cl} and $n_{Q,cl}$ elements. It is impossible to avoid the complicated intersections of recurrent survival events characterized by these elements. Therefore, the estimation of the long term safety of structures is connected with some methodological and mathematical improvements.

Improved computational methods are based on importance and conditional sampling procedures (Mori and Ellingwood 1993), direction simulation approaches (Ditlevsen 1997), the variable-complexity approach (Burton and Hajela 2003) and equivalent extreme-value events (Li *et al*. 2007). These improvements help researchers with reducing sophisticated computational procedures. However, they are inconvenient for structural designers and therefore not effective to translate them into everyday design reality.

The prediction of the long term survival probability of beam-columns may be based on the concept of transformed conditional probabilities (Kudzys and Lukoševičienė 2009; Kudzys *et al*. 2010). Since the model of a series system consists of equireliable and equicorrelated elements, the long term survival probability of a beam-column may be presented as

$$\mathbf{P}(T \geq t_n) = \mathbf{P}^n(S_k) \left[1 + \rho_{kl}^x \left(\frac{1}{\mathbf{P}(S_k)} - 1 \right) \right]^{n-1} , \quad (60)$$

where $\mathbf{P}(S_k)$ is defined by Eq. (58); ρ_{kl}^x is the correlation factor of a series system the bounded index of which is

$$x = \mathbf{P}(S_k) \left(\frac{4.5}{1 - 0.98\rho_{kl}} \right)^{0.5} \left[\frac{1 - \mathbf{P}^2(S_k)}{1 - \mathbf{P}^n(S_k)} \right]^{0.5\rho_{kl}} . \quad (61)$$

The coefficient of system element correlation is

$$\rho_{kl} = \frac{Cov(Z_k, Z_l)}{\boldsymbol{\sigma} Z_k \times \boldsymbol{\sigma} Z_l} = \frac{1}{1 + \boldsymbol{\sigma}^2 E_c / \boldsymbol{\sigma}^2 R_c} , \quad (62)$$

where $\boldsymbol{\sigma}^2 E_c$ is defined by Eqs. (54) and $\boldsymbol{\sigma}^2 R_c$ – by Eq. (52).

The prediction of the probabilistic reliabilities of beam-columns should be based on the reliability index approach. Their generalized reliability index may be introduced as

$$\beta(T \geq t_n) = \mathbf{\Phi}^{-1}\left[\mathbf{P}(T \geq t_n)\right], \quad (63)$$

where $\mathbf{\Phi}(\bullet)$ is the cumulative distribution function of standard normal distribution tabulated in statistic texts; $\mathbf{P}(T \geq t_n)$ is probability defined by Eq. (60). For persistent design situations during $t_n = 50$ years, the target value of reliability index $\beta_T(T \geq t_n)$ for the beam-columns of residential, office and public buildings is equal to 3.8 (EN 1990 2002).

When the action effects of beam-columns are provoked only by permanent loads, G, the resistance safety margin of these structural members may be expressed as $Z = R_C = \theta_R R - \theta_G E_G$. In this case, the reliability index of beam-columns may be defined as

$$\beta_G = \Phi^{-1}\left[\mathbf{P}(R_c > 0)\right] = R_{Cm}/\boldsymbol{\sigma} R_C, \quad (64)$$

where the statistics of their conventional resistance, R_C, is defined by Eqs. (51) and (52).

6. Numerical Illustration

6.1. The Parameters of Analysis

Consider the reliability indices β_N and β_M of the spun concrete beam-column of the braced two-storied frames of Reliability Class RC2 (Fig. 5) designed by directions EN 1990 (2002) and ASCE/SEI 7-05 (2006), (Kudzys and Kliukas 2009).

The means and variances of compressive forces and the first order bending moments are as follows:

$N_{Gm} = N_{Gk} = 612$ kN, $\boldsymbol{\sigma}^2 N_G = (0.1 \times 612)^2 = 3745$ (kN)^2;

$M_{OGm} = 28.8$ kNm, $\boldsymbol{\sigma}^2 M_{OG} = (0.1 \times 28.8)^2 = 8.29$ (kNm)^2;

$N_{Qm} = 0.47 N_{Qk} = 30.55$ kN, $\boldsymbol{\sigma}^2 N_Q = (0.58 \times 30.55)^2 = 314$ (kNm)^2;

$M_{OQm} = 7.64$ kNm, $\boldsymbol{\sigma}^2 M_{OQ} = (0.58 \times 7.64)^2 = 19.64$ (kNm)^2;

$N_{Sm} = N_{Sk}/(1 + k_{0.98}\boldsymbol{\delta} N_S) = 12.19$ kN;

$\boldsymbol{\sigma}^2 N_S = (0.5 \times 12.19)^2 = 37.2$ (kN)^2;

$M_{OSm} = 1.52$ kNm, $\boldsymbol{\sigma}^2 M_{OS} = (0.5 \times 1.52)^2 = 0.58$ (kNm)^2;

$N_{Em} = 612 + 30.55 + 12.2 = 654.75$ kN, $\boldsymbol{\sigma}^2 N_E = 3745 + 314 + 37.2 = 4096.2$ (kN)^2;

$M_{OEm} = 28.8 + 7.64 + 1.52 = 37.96$ kNm, $\boldsymbol{\sigma}^2 M_{OE} = 8.29 + 19.64 + 0.58 = 28.51$ (kNm)^2.

The geometrical parameters of the beam-column can be expressed as

$l_{om} = 3.0$ m, $\boldsymbol{\delta} l_o = 0.1$, $\boldsymbol{\sigma}^2 l_o = (0.1 \times 3.0) = 0.09$ m^2;

$r_2 = 0.15$ m, $r_{1m} = 0.09$ m, $r_s = 0.12$ m, $A_m = 0.04524$ m^2, $A_S = 0.00181$ m^2 (16$\varnothing$12), $A_{cm} = 0.0434$ m^2;

$\rho_m = 0.0417$, $I_m = \pi\left(r_2^4 - r_{1m}^4\right)/4 = 346 \times 10^{-6}$ m^4,

$I_s = \pi\left(r_{s2}^4 - r_{s1}^4\right)/4 = 13 \times 10^{-6}$ m^4,

$\boldsymbol{\delta} A_c \approx \boldsymbol{\delta} I = (1.2 - r_2)/\left[150(r_2 - r_{1m})\right] = 0.1167$,

$\boldsymbol{\sigma}^2 A_c = (0.1167 \times 0.0434)^2 = 25.65 \times 10^{-6}$ m^4,

$\boldsymbol{\sigma}^2 I = (0.1167 \times 346 \times 10^{-6})^2 = 0.00163 \times 10^{-6}$ m^8,

$\boldsymbol{\sigma}^2 \rho = \left(A_s / A_{cm}^2\right) \boldsymbol{\sigma}^2 A_c = 24.65 \times 10^{-6}$.

The parameters of spun concrete C50/60 are given by

$f_{ck} = 50$ MPa, $\alpha_2 = 0.85 - 1.7\rho_m = 0.779$,

$\alpha_{ccm} = 1 - 0.1 N_{Gm}/N_{Em} = 0.9065$, $f_{cm} = 58$ MPa,

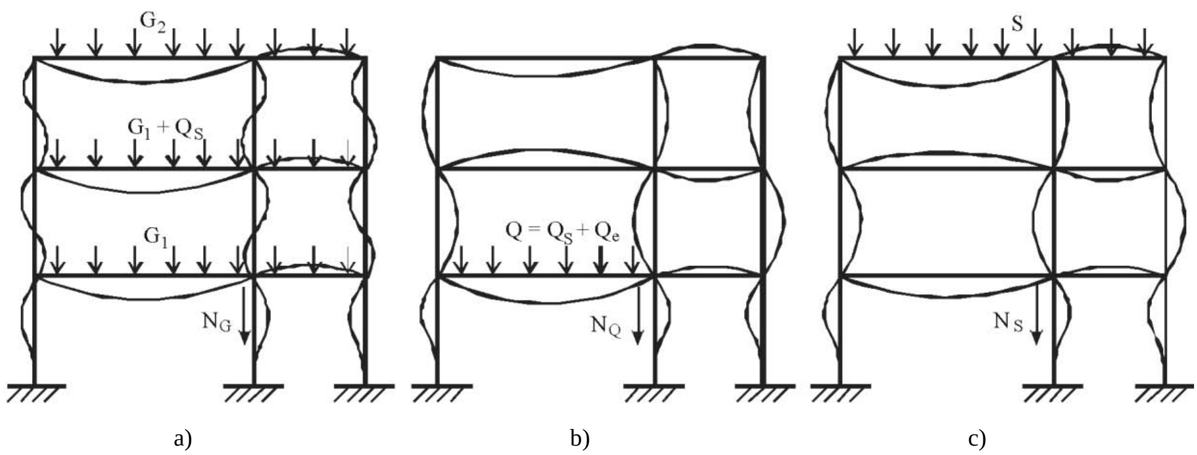

Fig. 5. Compressive forces N_G, N_Q and N_S caused by permanent (a) and variable extreme floor (b) or snow (c) loads

$\boldsymbol{\delta} f_c = 0.088 + 3(70 - f_{ck})^2 \times 10^{-5} = 0.10$,

$\boldsymbol{\delta} f_{cc} = (0.10^2 + 0.08^2)^{1/2} = 0.128$;

$f_{ccm} = \alpha_{ccm}\alpha_2 f_{cm} = 40.96$ MPa,

$\boldsymbol{\sigma}^2 f_{cc} = (\boldsymbol{\delta} f_{cc} \times f_{ccm})^2 = 27.49$ (MPa)^2;

$E_{cm} = 20\,(0.1 f_{cm})^{0.3} = 33.89$ GPa, $\boldsymbol{\sigma}^2 E_c = (0.15\times 33.89)^2 =$ 25.84 (GPa)^2.

The parameters of reinforcing high-strength cold worked bars are

$f_{0.2k} = 800$ MPa, $\sigma_{scm} = 452(1.36 + 4\rho_m) = 690$ MPa,

$\boldsymbol{\sigma}^2 \sigma_{sc} = (0.105\times 690)^2 = 5249$ (MPa),

$\boldsymbol{\delta} f_s = (0.09^2 + 0.12^2) = 0.15$, $f_{stm} = 500$ MPa,

$\boldsymbol{\sigma}^2 f_{st} = (0.15\times 500)^2 = 5625$ (MPa)^2, $f_{scm} = 600$ MPa,

$\boldsymbol{\sigma}^2 f_{sc} = (0.15\times 600) = 8100$ (MPa)^2.

The statistics of additional random variables are:
$\theta_{Nm} = \theta_{Mm} \approx 1.0$ and $\boldsymbol{\sigma}\theta_N = 0.05$, $\boldsymbol{\sigma}\theta_M \approx 0.10$ for action effects and $\theta_{R_N m} = 0.99$, $\boldsymbol{\sigma}\theta_{R_N} = 0.08$ and $\theta_{R_M m} = 1.0$, $\boldsymbol{\sigma}\theta_{R_M} = 0.14$ for the resistance of the compression and flexural members of the analysed annular cross sections.

6.2. Second Order Effects

According to Eqs. (25) and (26), the mean and variance of the stiffness factor are

$K_{cm} = 0.25/(1 + 1.7\times 28.8/37.96) = 0.1092$,

$$\boldsymbol{\sigma}^2 K = \left[(0.25\times 1.7\times 28.8)/(37.96 + 1.7\times 28.8)^2\right]^2 \times (8.29 + 28.51) = 9659 \times 10^{-8}.$$

Therefore, the statistics of the effective flexural stiffness of a beam column considering Eqs. (23) and (24) are

$$(EI)_m = 0.1092 \times 3.389 \times 10^4 \times 3.46 \times 10^{-4} + 2\cdot 10^5 \times 1.3 \times 10^{-5} = 3.88 \text{ MNm}^2,$$

$$\boldsymbol{\sigma}^2 (EI) = (0.1092 \times 3.389 \times 10^4)^2 \times 0.163 \times 10^{-8} + (0.1092 \times 3.46 \times 10^{-4})^2 \times 0.2584 \times 10^8 + (3.389 \times 10^4 \times 3.46 \times 10^{-4})^2 \times 9659 \times 10^{-8} = 0.07247 \text{ (MNm}^2)^2.$$

According to Eqs. (32) and (33), the mean and variance of buckling load are

$N_{Bm} = \pi^2 \times 3.88/3.0^2 = 4.255$ MN,

$$\boldsymbol{\sigma}^2 N_B = \pi^2 \times 0.07247/3.0^2 + (2\pi^2 \times 3.88/3.0^3)^2 \times 0.09 = 0.8037 \text{ (MN)}^2.$$

The first order mean value of the eccentricity of joint compressive force is

$$e_{om} = 37.96/654.75 = 0.058 \text{ m}.$$

Thus, the statistics of the second order eccentricity according to Eqs. (35) and (36) are

$$e_m = 0.058[1/(1 - 0.6547/4.255)] = 0.06855 \text{ m},$$

$$\boldsymbol{\sigma}^2 e = \left[\frac{0.6547\times 0.058}{(4.255 - 0.6547)^2}\right]^2 0.8037 + \left[\frac{4.255\times 0.058}{(4.255 - 0.6547)^2}\right] 4.096\times 10^{-3} = 8.384\times 10^{-6} \text{ m}^2.$$

6.3. Beam-Column as a Compression Member

The mean values of the response factors of member components taking into account Eqs. (2) and (3) are

$k_{cm} = 1 - 0.3\times 0.06855/(0.12\times 1.417) = 0.8791$,

$k_{sm} = 1 - 0.34\times 0.06855/0.12 = 0.8058$.

Then, according to Eqs. (6) and (7), the mean and variance of beam-column resistance are

$$R_{Nm} = N_{Rm} = (0.8791\times 0.0434\times 40.96 + 0.8058\times 0.00181\times 690)0.12 / (0.06855 + 0.12) = 1.6351 \text{ MN},$$

$$\boldsymbol{\sigma}^2 R_N = (0.8791\times 0.0434\times 0.6364)^2\times 27.49 + (0.8791\times 40.96\times 0.6364)^2\times 25.65\times 10^{-6} + (0.8058\times 0.00181\times 0.6364)^2\times 5249 + (40.96\times 0.0434\times 1.2117 + 1.34\times 690\times 0.00181)3.3754)^2\times 8.384\times 10^{-6} + (1.6782 + 0.9656)^2\times 24.65\times 10^{-6} + 2(0.1161\times 0.01488 + 0.06725\times 0.01488) = 0.04122 \text{ (MN)}^2.$$

According to Eqs. (51)–(54), the means and variances of conventional resistance, R_{CN}, and extreme action effects are

$R_{CNm} = 0.99\times 1.6351 - 1.0\times 0.612 = 1.0067$ MN,

$$\boldsymbol{\sigma}^2 R_{CN} = 0.99^2\times 0.04122 + 1.6351^2\times 0.0064 + 1.0^2\times 3745\times 10^{-6} + 0.612^2\times 0.0025 = 0.06222 \text{ (MN)}^2;$$

$N_{Qm} = 1.0\times 0.03055 = 0.03055$ MN,

$\boldsymbol{\sigma}^2 N_Q = 1.0^2\times 3.14\times 10^{-6} + 0.03055^2\times 0.0025 = 316\times 10^{-6}$ (MN)^2;

$N_{Sm} = 1.0\times 0.0122 = 0.0122$ MN,

$\boldsymbol{\sigma}^2 N_S = 1.0^2\times 37.2\times 10^{-6} + 0.0122^2\times 0.0025 = 38\times 10^{-6}$ (MN)^2;

$N_{QSm} = 0.03055 + 0.0122 = 0.04275$ MN,

$\boldsymbol{\sigma}^2 N_{QS} = 316\times 10^{-6} + 38\times 10^{-6} = 354\times 10^{6}$ (MN)^2.

According to Eq. (62), the coefficients of the correlation of the cuts of safety margins Z_{QN}, Z_{SN} and Z_{QSN} are $\rho_Q = 0.99495$, $\rho_S = 0.9994$ and $\rho_{QS} = 0.99434$.

The reliability parameters of beam-columns are presented in Table 1. Because coefficients $\rho_Q \approx \rho_S \approx \rho_{QS} \approx 1$, index $\beta_N = 3.78$ may be treated as the basic parameter the value of which is close to target reliability index $\beta_T = 3.80$ (EN 1990 2002).

6.4. Beam-Column as a Bending Member

According to Eqs. (15), (16) and (17), the means of bending resistance components are

$$T_{1m} = 0.0434\times40.96+0.0181(500+600)=3.7687 \text{ MN},$$

$$T_{2m} = 1.2\times0.12(0.00181\times500+0.6547)=0.2246 \text{ MNm},$$

$$T_{3m} = 0.0434\times40.96+0.00181\times600–0.6547=2.209 \text{ MN}.$$

Thus, according to Eqs. (14) and (18), the mean and variance of beam-column resistance are

$$R_{Mm} = M_{Rm} = 0.2246\times 2.209/3.7687 = 0.1316 \text{ MNm},$$

$$\boldsymbol{\sigma}^2 R_m = \left[0.2246(3.7687-2.209)/3.7687^2\right]^2 \times (0.0434^2\times27.49+40.96^2\times25.65\times10^{-6}+0.00181^2\times 8100)+ [0.00181\times2.209(1.2\times0.12\times 3.7687–0.2246) / 3.7687^2]^2\times5625+[(1.2\times0.12\times 2.209–0.2246) / 3.7687]^2\times4096\times10^{-6}= 121.4\times10^{-6} \text{ (MNm)}^2.$$

The statistics of the second order bending moment components from Eqs. (53) and (54) are

$$M_{Gm} = 1.0\times0.612\times0.06855=0.04195 \text{ MNm};$$

$$\boldsymbol{\sigma}^2 M_G = (0.10\times0.04195)^2=17.6\times10^{-6} \text{ (MNm)}^2,$$

$$\boldsymbol{\sigma}^2(\theta_M M_G) = 1.0^2\times17.6\times10^{-6}+0.04195^2\times0.01= 35.2\times10^{-6} \text{ (MNm)}^2;$$

$$M_{Qm} = 1.0\times0.03055\times0.06855=0.002094 \text{ MNm},$$

$$\boldsymbol{\sigma}^2 M_Q = (0.58\times0.002094)^2=1.475\times10^{-6} \text{ (MNm)}^2,$$

$$\boldsymbol{\sigma}^2(\theta_M M_Q) = 1.0^2\times1.475\times10^{-6}+0.002094^2\times0.01= 1.518\times10^{-6} \text{ (MNm)}^2;$$

$$M_{Sm} = 1.0\times0.0122\times0.06855=0.000836 \text{ MNm},$$

$$\boldsymbol{\sigma}^2 M_S = (0.50\times0.000836)^2=0.174\times10^{-6} \text{ (MNm)}^2,$$

$$\boldsymbol{\sigma}^2(\theta_M M_S) = 1.0^2\times0.174\times10^{-6}+0.000836^2\times0.01= 0.181\times10^{-6} \text{ (MNm)}^2;$$

$$M_{QSm} = 1.0\times0.002094+0.000836=0.00293 \text{ MNm},$$

$$\boldsymbol{\sigma}^2(\theta_M M_{QS}) = 1.518\times10^{-6}+0.181\times10^{-6}=1.699\times10^{-6} \text{ (MNm)}^2.$$

According to Eq. (62), the coefficients of the correlation of the cuts of safety margins Z_{QM}, Z_{SM} and Z_{QSM} are $\rho_Q = 0.99697$, $\rho_S = 0.99964$ and $\rho_{QS} = 0.99661$.

The reliability parameters of beam columns are presented in Table 2. The reliability indices are very close to their values given in Table 1. Because the eccentricity ratio of compressive force $e/r_s = 0.06855/0.12 = 0.57<1$, the considered beam-column may be treated as a compression member the generalized reliability index of which is equal to 3.78 (see Table 1).

It is called reader's attention to the values of the reliability indices of beam-columns exposed only to permanent compressive forces and bending moments. According to Eq. (64), the reliability indices of the considered beam-columns are equal to

$$\beta_{GN} = 1.0067/\sqrt{0.06222} = 4.04,$$

$$\beta_{CM} = 0.08965/\sqrt{0.00469} = 4.03.$$

Table 1. Parameters of beam-columns as compression members

R_{CNm} by (51), MN	$\boldsymbol{\sigma}^2 R_{CN}$ by (52), (MN)^2	Extreme actions	Recurrent number n by (45)	N_{Cm} by (53), MN	$\boldsymbol{\sigma}^2 N_C$ by (54), (MN)^2	$\mathbf{P}(R_{CN} > N_C)$ by (58)	$\mathbf{P}(T \geq t_n)$ by (60)	$\beta_N(T \geq t_n)$ by (63)
1.0067	0.068 60	Q	50	0.030 55	316×10^{-6}	0.999 952	0.999 921	3.78
		S	50	0.012 20	38×10^{-6}	0.999 966	0.999 963	3.97
		Q + S	4.25	0.042 75	354×10^{-6}	0.999 942	0.999 932	3.82

Table 2. Parameters of beam-columns as bending members

R_{CMm} by (51), MNm	$\boldsymbol{\sigma}^2 R_{CM}$ by (52), (MNm)^2	Extreme actions	Recurrent number n by (45)	M_{Cm} by (53), MN	$\boldsymbol{\sigma}^2 M_C$ by (54), (MN)^2	$\mathbf{P}(R_{CM} > M_C)$ by (58)	$\mathbf{P}(T \geq t_n)$ by (60)	$\beta_M(T \geq t_n)$ by (63)
0.089 65	0.000 496	Q	50	0.002 09	1.518×10^{-6}	0.999 957	0.999 938	3.84
		S	50	0.000 836	0.181×10^{-6}	0.999 967	0.999 965	3.98
		Q + S	4.25	0.002 93	1.699×10^{-6}	0.999 949	0.999 944	3.86

These values exceed the generalized reliability index $\beta_N(T \geq t_n) = 3.78$ not much. It shows that the variable actions of residential, office and public low rise buildings with reinforced concrete floors may only slightly decrease the structural safety of columns.

7. Conclusions

The resistance concept of spun concrete beam-columns may be based on their ultimate resisting compressive force or resisting bending moment values. Therefore, the structural safety of these members may be assessed and predicted by the generalized reliability index $\beta_N(T \geq t_n)$ or $\beta_M(T \geq t_n)$ from Eq. (63) respectively.

The non-stationary time-dependent resistance safety margin of building beam-columns is closely related to their action effects provoked both by live floor and climate actions. Instead of the sustained and extraordinary components of live floor loads, it is expedient to use their annual extreme sums in probabilistic design practice. The mean and variance of joint annual extreme action effects are expressed by Eqs. (38) and (40). Live and climate (wind and snow) annual extreme actions effects modelled by Type 1 (Gumbel) distribution help engineers with expressing the resistance safety margin of building beam-columns as a stationary process.

The reliability levels of beam-columns designed by limit state and probability-based approaches were compared. Regardless of methodological features, both limit state design methods EN 1990 and ASCE/SEI 7-05 lead to the same design results and are confirmed by the reliability index values. The reliability index approach based on the transformed conditional probability concept opens quite realistic design formats in the long term structural safety prediction of beam-columns and other structural members.

References

ACI Committee 318-05. 2005. Building code requirements for structural concrete. American Concrete Institute, Farmington Hills, Mich.

ASCE/SEI 7-05. 2006. Minimum design loads for buildings and other structures. American Society of Civil Engineers, Structural Engineering Institute, Reston, Virginia.

Burton, S. A.; Hajela, P. A. 2003. A variable-complexity approach to second-order reliability-based optimization, *Structural Optimization* 25(4): 237–250. doi:10.1007/s00158-003-0298-3

Ditlevsen, O. 1997. Structural reliability codes for probabilistic design – a debate paper based on elementary reliability and decision analysis concepts, *Structural Safety* 19(3): 253–270. doi:10.1016/S0167-4730(97)00009-X

Ellingwood, B. R. 1981. Wind and snow load statistics for probabilistic design, *Journal of Structural Division* 107(7): 1345–1349.

EN (1990) 2002. Eurocode – Basis of structural design. Brussels.

EN (1992-1) 2004. Eurocode 2: Design of concrete structures – Part 1: General rules and rules for buildings. Brussels.

ISO (2394) 1998. General principles on reliability for structures. 2nd ed. Switzerland.

Jankovski, V.; Atkočiūnas, J. 2008. Matlab implementation in direct probability design of optimal steel trusses, *Mechanika* (6): 30–37.

JCSS. 2000. Probabilistic model code: Part 1 – Basis on design. Joint Committee on Structural Safety.

Juocevičius, V.; Vaidogas, E. R. 2010. Effect of explosive loading on mechanical properties of concrete and reinforcing steel: towards developing a predictive model, *Mechanika* (1): 5–12.

Kargaudas, V.; Adamukaitis, N. 2010. Post-elastic force-displacement dependence of bent and compressed column, *Mechanika* (3): 5–9.

Kliukas, R.; Kudzys, A.; Vadlūga, R. 2010. The quasi-ductile resistance of spun concrete beam-columns, *Journal of Civil Engineering and Management* 16(2): 189–196. doi:10.3846/jcem.2010.20

Kudzys, A.; Kliukas, R. 2009. The limit state design of spun concrete columns using load combinations recommended by EN 1990 and ASCE/SEI 7-05, *Journal of Civil Engineering and Management* 15(3): 289–297. doi:10.3846/1392-3730.2009.15.289-297

Kudzys, A.; Lukoševičienė, O. 2009. On the safety prediction of deteriorating structures, *Mechanika* (4): 5–11.

Kudzys, A.; Kliukas, R.; Vadlūga, R. 1993. Utilization of high-strength spun concrete and reinforcing steel in compressive structures, in *Proceedings High-Strength Concrete, Norway* 1: 259–268.

Kudzys, A.; Lukoševičienė, O.; Baltrukėnaitė-Kroškienė, I. 2010. Structural safety of rolled and welded beams subjected to lateral-torsional buckling, *Journal of Civil Engineering and Management* 16(2): 172–178. doi:10.3846/jcem.2010.18

Li, J.; Chen, J.; Fan, W. 2007. The equivalent extreme-value event and evaluation of the structural system reliability, *Structural Safety* 29: 112–131. doi:10.1016/j.strusafe.2006.03.002

MacGregor, J. G. 1988. *Reinforced concrete, mechanics and design*. Practice-Hall International Inc.

Mori, V.; Ellingwood, B. 1993. Reliability based service-life assessment of aging concrete structures, *Journal of Structural Engineering* ASCE, 119(5): 1600–1621. doi:10.1061/(ASCE)0733-9445(1993)119:5(1600)

Ramsay, R. J.; Mirza, S. A.; Macgregor, J. G. 1979. Monte Varlo study of shorttime deflections of reinforced concrete beams, *Journal of the ACJ* 76(8): 897–918.

Rosowsky, D.; Ellingwood, B. 1992. Reliability of wood systems subjected to stochastic live loads, *Wood and Fiber Science* 24(1): 47–59.

Vaidogas, E. R.; Juocevičius, V. 2008. Reliability of a timber structure exposed to fire: estimation using fragility function, *Mechanika* (5): 35–42.

Vrouwenvelder, A. C. W. M. 2002. Developments towards full probabilictic design codes, *Structural Safety* 24(2–4): 417–432. doi:10.1016/S0167-4730(02)00035-8

Židonis, I. 2009. Method for a direct calculation of stress-strain state parameters at normal right-angled sections of structural members given curvilinear stress diagrams, *Mechanika* (3): 27–33.

Вадлуга, Р. 1979. О практическом расчете по прочности бетонов и железобетонных элементов кольцевого сечения [Vadluga, R. Particial method of strength anglysis of concrete and reinforced concrete ring cross-section], *Железобетонные конструкции* [Concrete Structures] 9: 49–58.

TIKIMYBINIS SIJINIŲ KOLONŲ IŠ CENTRIFUGUOTOJO BETONO PROJEKTAVIMAS

A. Kudzys, R. Kliukas

Santrauka

Atkreiptas inžinierių dėmesys į didžiastiprio plieno strypais armuotų sijinių kolonų iš centrifuguotojo betono naudojimą įstaigų ir administraciniuose pastatuose. Aptarta galimybė projektuojant pastato rėmo sijines kolonas taikyti patikimumo indekso būdą. Analizuojamas sijinių kolonų kaip lenkiamai gniuždomų ir gniuždomai lenkiamų žiedinio skerspjūvio elementų atspario kriterijus. Nagrinėjamos sijinių kolonų pirmosios ir antrosios eilės įrąžos. Pateiktas laike kintantis ribinės saugos atsparis ir jo stacionarusis ekvivalentas. Tyrinėjamas nesudėtingas taikomasis modelis rėmo sijinių kolonų tikimybiniam projektavimui. Skaitiniu pavyzdžiu iliustruojamas stabilizuojamų rėmų sijinės kolonos projektavimas.

Reikšminiai žodžiai: centrifuguotasis betonas, rėmai, antrosios eilės įrąžos, ribinė atspario sauga, patikimumo indekso skaičiavimas.

Antanas KUDZYS. Prof., Dr Habil at the Institute of Architecture and Construction, Kaunas University of Technology, Tunelio g. 60, LT-44405 Kaunas, Lithuania. Dr Habil (building structures, 1967). The author of 6 books, 5 course books for students, 4 dictionaries and more than 300 articles. Research interests include bearing capacity, safety and durability of structural members and their systems.

Romualdas KLIUKAS. Prof., Dr at the Department of Strength of Materials, Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-10223 Vilnius, Lithuania. PhD in building structures, 1989. A researcher at the Department of Reinforced Concrete Structures, Vilnius Institute of Civil Engineering. Research visit to Building Scientific-Technical Research Centre (France) in 1990-91. The author of more than 50 articles. Research interests include capacity, durability and renovation of reinforced vibrated and spun concrete structures, safety and durability of structural members and their systems.