Supplementary information

DETERMINING CONCESSIONARY ITEMS FOR "AVAILABILITY PAYMENT ONLY" PPP PROJECTS: A HOLISTIC FRAMEWORK INTEGRATING VALUE-FOR-MONEY AND SOCIAL VALUES

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Appendix 1. Calculation process explanation for the coefficient of social value

The estimation of the coefficient of the CD function model is mainly based on various regression models (Kleyn *et al.* 2017). The use of regression methods requires a corresponding sample size. However, the case project used in this study is a portfolio of municipal roads, bridges, and parks. Samples of similar portfolio projects are not readily available. Therefore, we adopted a simplified approach. The steps are as follows:

1) *Assumed elasticity coefficients a and b based on the empirical estimation.*

The operating costs are invested in stages during the long-term operation period, while the construction investment invests entirely in the first few years. The increase in operating input has a more pronounced effect on the social benefits (acquired by end users), assuming that the elasticity of operating input is greater than that of construction investment. Assuming constant returns to scale, i.e., taking $a + b = 1$, we assumed $b = 0.6$, $a = 0.4$.

2) *Determining λ*

We collected four similar projects and investigated the consult's experience to estimate the social value of each project. We can calculate the coefficient λ for each similar project according to the project information and consult's estimation. Then, we adopted the average value to serve as the coefficient λ of the case project in this article.

3) *Testing the result of estimated λ*

The coefficient λ represents the contribution of technology and management level improvement other than elements to output. For PPP projects, the general view is that the introduction of private sectors can use their managerial and technical advantages; thus, the value of coefficient λ should be greater than 1. Secondly, the project involves roads, bridges, and parks, which belong to traditional infrastructure, and their construction and operation are relatively mature; therefore, the coefficient λ will not be too large. Moreover, the consulting manager of our case project estimated that the social value is about RMB 220 million. The estimation error is acceptable compared to the model calculation result, RMB 228.89 million. Overall, we believe the estimated λ equals 1.5 is reasonable and applicable.

Besides, the author has to point out that estimating social value is a complex systematic work. Although the estimation in this study is rough, considering we are using similar projects for estimating and adopting professional opinions for verifying, the estimation of this study can be used.

Table S1. Estimation of coefficient λ

Project No.	Total construction investment [1] (unit: million RMB)	Total operation cost [1] (unit: million RMB)	Project description	Social value [2] (unit: million RMB)	Estimated λ
P1	243.54	4.47	Municipal roads and bridges	40	1.8091
P2	903.33	91.89	Municipal roads, parks, and schools	240	1.0469
P3	2981.23	165.35	Municipal roads	950	1.8069
P4	765.00	21.42	Municipal roads and parks	120	1.3402
Avg.	1223.28	70.78	–	337.5	1.5008

Note: [1] Data source: Execution scheme of each project; [2] Data source: estimation of consulting manager in charge of each project.

Appendix 2. Detailed solution process of set ψ based on the game result

According to Eqn (18), Figure 6 shows the solution set, i.e., set Ψ, of concessionary items based on the game result. Through the game process, we can get the V_G^* and V_I^*. According to Eqn (2), since V_G is a function of three variables, i.e., (r_α, r_m, t_2), a series of combinations of (r_α, r_m, t_2) allow us to arrive at a specific value of V_G. This logic means that V_G^* also corresponds to a series of (r_α, r_m, t_2) combinations. Equation $V_G(r_\alpha, r_m, t_2) = V_G^* = 56.35$ theoretically has an infinite number of solutions. Considering the background of the practical problem, we adopt the discretization approximation method to obtain a set of practical solutions. The solution steps are as follows:

1) Considering the feasible region according to the practical background:

$$\Phi = \left[\left(r_\alpha, r_m, t_2 \right) \middle| 5\% \le r_\alpha \le 12\%;\ 5\% \le r_m \le 24\%;\ 10 \le t_2 \le 30 \right];$$

2) Discretizing r_α and r_m by one basis point (0.01%);
3) Discretizing t_2 by one year;
4) Calculating the V_G value of each point in the variable matrix of 2031*2031*31.
5) Comparing those values with V_G^* to find approximate solutions

The program codes used in the solution steps are as follows:

```
clear
% Discretizing Ra and Rm by one basis point (0.01%);
ralower = 0.05;
raupper = 0.12;
ranum = ceil((raupper-ralower) / 0.001);
    ra = zeros(1,ranum);
for h = 1:1:ranum
ra(1,h) = ralower + 0.001*h;
end
rmlower = 0.05;
rmupper = 0.24;
rmnum = ceil((rmupper-rmlower) / 0.001);
rm = zeros(1,rmnum);
for h = 1:1:rmnum
    rm(1,h) = rmlower + 0.001*h;
end
% Discretizing T by one year;
t2lower = 10;
t2upper = 30;
t2num = ceil(t2upper-t2lower + 1);
t2 = zeros(1,t2num);
for h = 1:1:t2num
    t2(1,h) = t2lower + h-1;
end
% Calculating the VG value of each point in the variable matrix of 2031*2031*31
vgcau = zeros(ranum,rmnum,2);
for m = 1:1:t2num
    for h = 1:1:ranum
        for g = 1:1:rmnum
        vgcau(h,g,m) = a3(ra(1,h),rm(1,g),t2(1,m));
        end
    end
end
logic = (abs(vgcau-vgstar))/vgstar <= 0.001;
a = vgcau(logic).
```

Reference

Kleyn, J., Arashi, M., Bekker, A., & Millard, S. (2017). Preliminary testing of the Cobb–Douglas production function and related inferential issues. *Communications in Statistics – Simulation Computation*, *46*(1), 469–488. https://doi.org/10.1080/03610918.2014.968724