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A MULTI-PERIOD NEWSVENDOR PROBLEM WITH PRE-SEASON EXTENSION UNDER FUZZY DEMAND

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Abstract. This paper proposes a fuzzy multi-period newsvendor model with pre-season extension for innovative products. The demand of the product is represented by fuzzy numbers with triangular membership function. The holding and shortage cost parameters are considered as imprecise and also represented by triangular fuzzy numbers. As the selling season draws closer, suppliers lead times shortens and thus production costs increase. In contrast, caused by the oncoming selling season, demand fuzziness decreases and more accurate demand forecasts can be maintained that lead to lower overage/underage costs. The objective of the model is to find the best order period and the best order quantity that will minimize the fuzzy expected total cost. The model is experimented with an illustrative example and supported by sensitivity analyses.

Keywords: inventory problem, fuzzy modeling, newsvendor, innovative product, fuzzy demand, fuzzy inventory cost, pre-season.

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1. Introduction

In today's competitive market conditions, development of innovative products is extremely important for an enterprise to achieve and sustain the superiority. Innovative products are products that have shorter life cycles with higher innovation and fashion contents (Lee 2002). Fashion goods, electronic products and mass customized goods are examples of innovative products. Although innovative products tend to have higher product profit margins, the cost of obsolescence is high for them. These kinds of products have much product variety and short product life cycles. Caused by the innovative nature of the product, usually no historical data are available for forecasting demand of such products. In addition to the short life cycle and high demand unpredictability of these products, another important feature is the long supply lead times. Because of having such long supply lead times and short sales seasons, procurement problem of these products corresponds to the single-period inventory (also known as newsvendor or newsboy) problem.

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The single-period inventory problem (SPP) deals with finding product's order quantity that minimizes the expected total cost or maximizes the expected total profit under linear purchasing, holding, and shortage costs and probabilistic demand. In SPP, product orders are given before the selling period begins. There is no option for an additional order during the selling period or there will be a penalty cost for this re-order. The assumption of the SPP is that if any inventory remains at the end of the period, either a discount is used to sell it or it is disposed of (Nahmias 1996). If the order quantity is smaller than the realized demand, the seller misses some profit. An extensive literature review on a variety of extensions of the single-period inventory problem and related multi-stage, inventory control models can be found Khouja (1999) and Silver *et al.* (1998). Most of the extensions have been made in the probabilistic framework, in which the uncertainty of demand is described by probability distributions. However, the methods based on the probability theory allow only quantitative uncertainties. For instance, Kopytov *et al.* (2006) developed a stochastic single-product inventory control model for the chain "producer – wholesaler – customer" for transport and industrial companies. The optimization criterion is the minimum of average expenses for goods holding, ordering and losses from deficit per a time unit.

In reality, most of the evaluations are imprecise, fuzzy and cannot be quantified. The fuzzy set theory, introduced by Zadeh (1965), is the best form that adapts all the uncertainty set to the model. When subjective evaluations are considered, the possibility theory takes the place of the probability theory (Zadeh 1978). The fuzzy set theory can represent linguistic data which cannot be easily modeled by other methods (Dubois and Prade 1980).

In the literature, fuzzy modeling is used for single-period inventory problems by several researchers. Ishii and Konno (1998) introduced a fuzzy newsboy model restricted to shortage cost that is given by an L-shape fuzzy number while the demand is still stochastic. Thus the total expected profit function also becomes a fuzzy number and an optimum order quantity is obtained in the sense of fuzzy max (min) order of the profit function. Petrovic *et al.* (1996) developed two fuzzy models to handle uncertainty in the SPP. In the first model, uncertain demand is represented by a discrete fuzzy set but the inventory costs thought as precise and in the second model inventory costs also described as triangular fuzzy numbers. In the paper the concept of level-2 fuzzy set and the method of arithmetic defuzzification are employed to access an optimum order quantity. Li *et al.* (2002) studied the single-period inventory problem in two different cases to maximize the profit through ordering fuzzy numbers with respect to their total integral values. In order to minimize the fuzzy total cost, Kao and Hsu (2002) constructed a single-period inventory model. They adopted a method for ranking fuzzy numbers to find the optimum order quantity in terms of the cost. In their first study, Dutta *et al.* (2005) presented a SPP in an imprecise and uncertain mixed environment. They introduced demand as a fuzzy random variable and developed a new methodology to determine the optimum order quantity where the optimum is achieved using a graded mean integration representation. In their second study, Dutta *et al.* (2007) proposed a single-period inventory model of profit maximization with a reordering strategy in an imprecise environment. They divided the entire ordering period into two slots and considered the customer demand as a fuzzy number in both slots. They represented a solution procedure using ordering of fuzzy numbers with respect to their possibilistic mean values. Ji and Shao (2006) extended SPP in bi-level context, where the decision of retailer (lower level) was considered separate from this of the "manufacturer" (upper level). In another study Shao and Ji (2006) extended SPP in multi-product case with fuzzy demands under budget constraint. In both studies they adopt credibility theory and used chance-constrained programming. They solved their models by a hybrid intelligent algorithm based on genetic algorithm and developed a fuzzy simulation. Lu (2008) studied a fuzzy newsvendor problem to analyze optimum order policy based on probabilistic fuzzy sets with hybrid data. They verified that the fuzzy newsvendor model is one extension of the crisp models. Xu and Zhai (2008) proposed a fuzzy model to find an optimal technique for dealing with the fuzziness aspect of demand uncertainties. They used a triangular fuzzy number to model the external demand, and they developer optimal decision models for both an independent retailer and an integrated supply chain. In their more recent study, Xu and Zhai (2010) considered a two-stage supply chain coordination problem and focused on the fuzziness aspect of demand uncertainty. They used fuzzy numbers to depict customer demand, and investigated the optimization of the vertically integrated two-stage supply chain under perfect coordination and contrast with the non-coordination case.

Most of the papers mentioned above proposed single-period inventory models with only fuzzy environment extensions. Through this paper, we combine two extensions to SPP which are; an extension to fuzzy environment and an extension to models with more than one period to prepare for the selling season. The idea behind the second extension is that there may be many periods to produce or purchase the items which will be sold in a single season. The question becomes when and what orders should be placed from the suppliers as the selling season draws closer. Related references about this extension can be found in Silver *et al.* (1998). However, these papers consider only the probabilistic demand cases. In this study, a multi-period newsvendor problem with pre-season extension under fuzzy demand is analyzed. As the selling season draws closer, suppliers lead times shortens and the production cost increases. In contrast, demand fuzziness decreases and more accurate demand forecasts can be maintained that lead to lower overage/underage costs. The purpose of this study is to find the best order period and best order quantity for the single-period inventory problem with multi-period pre-season extension under fuzzy demand.

The remainder of the paper is organized as follows. The single-period inventory problem is subjected in Section 2. In Section 3, a fuzzy model with multi-period pre-season extension is described. A numerical illustration of the presented model is issued and sensitivity analyses over cost parameters are presented. Lastly the paper is concluded in Section 4.

2. Single-period inventory problem

The objective of the stochastic single-period inventory problem is to determine the order quantity Q^* for a fixed time period that will minimize the expected total cost. The total cost is the linear sum of purchase, overage, and underage costs. It is assumed that there is no initial inventory on hand. Demand is a random variable and represented by probability distributions. Items are purchased (or produced) for a single-period at the cost of c_p. The holding cost which is the cost of storing excess products minus their salvage value is c_h and the shortage cost which is the cost of lost sales due to the inability to supply the demand is c_s. The total cost function will be as follows:

$$TC(Q,X) = c_p Q + c_h \max\{(Q-X),0\} + c_s \max\{(X-Q),0\}, \tag{1}$$

where Q represents order quantity and X stands for the demand given by the domain $X = \{x_0, x_1, x_2, ..., x_n\}$. If demand has a probability distribution function $p_X(x_i)$, than the total expected cost function will be:

$$\begin{aligned} &E[TC(Q,X)] = \sum_{i=0}^{n} TC(Q,x_i)\, p(x_i) = \\ &c_p Q + \sum_{i=0}^{n} \left[c_h \max\{(Q-x_i),0\} + c_s \max\{(x_i-Q),0\} \right] p(x_i), \\ &(i = 0,1,2,...,n) \end{aligned} \tag{2}$$

which is equal to:

$$E[TC(Q,X)] = c_p Q + \sum_{i=0}^{Q-1} \left[c_h (Q-x_i)\, p(x_i) \right] + \sum_{i=Q}^{n} \left[c_s (x_i - Q)\, p(x_i) \right]. \tag{3}$$

Let

$$\Delta E[TC(Q,X)] = E[TC(Q+1,X)] - E[TC(Q,X)]. \tag{4}$$

Then, $\Delta E[TC(Q,X)]$ is the change in expected total cost when we switch from Q to $Q + 1$. For a convex cost function, the optimum Q will be the lowest Q where $\Delta E[TC(Q,X)]$ is greater than zero. Therefore, we select the smallest Q for which,

$$\Delta E[TC(Q,X)] \geq 0. \tag{5}$$

The equation above holds if

$$E[TC(Q+1,X)] - E[TC(Q,X)] \geq 0. \tag{6}$$

Substituting Equation 3 into Equation 6 leads to:

$$\begin{aligned} &c_p (Q+1) + \sum_{i=0}^{Q} \left[c_h (Q+1-x_i)\, p(x_i) \right] + \sum_{i=Q+1}^{n} \left[c_s (x_i - Q - 1)\, p(x_i) \right] - \\ &\left[c_p (Q) + \sum_{i=0}^{Q-1} \left[c_h (Q-x_i)\, p(x_i) \right] + \sum_{i=Q}^{n} \left[c_s (x_i - Q)\, p(x_i) \right] \right] \geq 0 \\ &c_p + \sum_{i=0}^{Q} \left[c_h p(x_i) \right] - \sum_{i=Q+1}^{n} \left[c_s p(x_i) \right] \geq 0. \end{aligned} \tag{7}$$

While $\sum_{i=0}^{n} p_X(x_i) = 1$,

$$p_{X\leq}(Q) \geq \frac{c_s - c_p}{c_h + c_s}, \tag{8}$$

where $p_{X\leq}(Q)$ is the probability that the demand X is smaller or equal to the order quantity Q. The expected total cost, $E\left[TC(Q,X)\right]$ will be minimized by the smallest value of Q(call it Q^*) satisfying the equation above (Equation 8).

3. Fuzzy model with multi-period pre-season extension

The fuzzy set theory provides a proper framework for description of uncertainty related to vagueness of natural language expressions and judgments. Fuzzy sets have been introduced by Zadeh (1965) as an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition where an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1].

Let U be a classical set of objects, called the universe, whose generic elements are denoted by x. Membership in a classical subset X of U is often viewed as a characteristic function, μ_X from U to such that:

$$\mu_X(x) = \begin{cases} 1 & \text{iff} \quad x \in X \\ 0 & \text{iff} \quad x \notin X \end{cases}. \tag{9}$$

If the valuation set ({0,1}) is allowed to be the real interval [0,1], $\widetilde{X}$ is called a fuzzy set (Zadeh 1965), $\mu_{\widetilde{X}}(x)$ is the grade of membership of x in $\widetilde{X}$. The closer the value of $\mu_{\widetilde{X}}(x)$ is to, the more x belongs to $\widetilde{X}$. $\widetilde{X}$ is completely characterized by the set of pairs:

$$\widetilde{X} = \{(x, \mu_{\widetilde{X}}(x)),\ x \in \widetilde{X}\}. \tag{10}$$

Fuzzy numbers are a particular kind of fuzzy sets. A fuzzy number is a fuzzy set R of real numbers set with a continuous, compactly supported, and convex membership function.

Let U be a universal set; a fuzzy subset $\widetilde{X}$ of U is defined by a function $\mu_{\widetilde{X}}(x): U \to [0,1]$ is called membership function. Here, U is assumed to be the set of real numbers R and $F$is the space of fuzzy sets.

The fuzzy set $\widetilde{X} \in F$ is a fuzzy number iff:

I. For $\forall\alpha \in [0,1]$, the set $X^\alpha = \{x \in R : \mu_{\widetilde{X}}(x) \geq \alpha\}$, which is called α cut of $\widetilde{X}$ is a convex set.

II. $\mu_{\widetilde{X}}(x)$ is a continuous function.

III. $\sup(\widetilde{X}) = \{x \in R : \mu_{\widetilde{X}}(x) \geq 0\}$ is a bounded set in R.

IV. $height(\widetilde{X}) = \max_{x \in U} \mu_{\widetilde{X}}(x) = h \geq 0$.

By conditions (I) and (II), each α-cut is a compact and convex subset of R hence it is a closed interval in R, $X^\alpha = \left[X_L(\alpha), X_R(\alpha)\right]$. If $h = 1$ we say that the fuzzy number is normal.

Let us show a fuzzy number $\widetilde{X}=(x_l, x_m, x_n, x_u)$ where $x_l < x_m < x_n < x_u$ in this form. The fuzzy number $\widetilde{X}$ is a so-called triangular fuzzy number $\widetilde{X}=(x_l, x_m = x_n, x_u)$, $x_l < x_m = x_n < x_u$ if its membership function $\mu_{\widetilde{X}}(x): R \rightarrow [0,1]$ is equal to (Fig. 1):

$$\mu_{\widetilde{X}}(x)=\begin{cases} 0 & x < x_l \\ \left(\dfrac{x-x_l}{x_m-x_l}\right) & x_l<x<x_m \\ \left(\dfrac{x_u-x}{x_u-x_m}\right) & x_m<x<x_u \\ 0 & x>x_u \end{cases}. \tag{11}$$

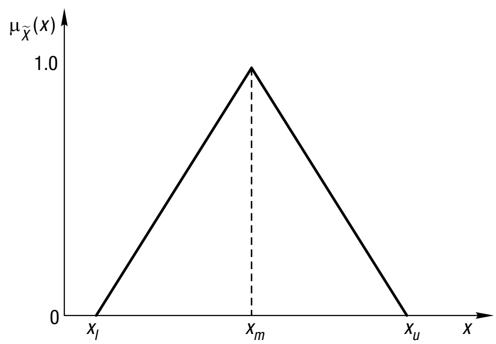

Fig. 1. Triangular membership function

The fuzzy function $\widetilde{X}$ is a function of parameters x_l, x_m and x_u, known as the lower value, middle value and upper value respectively. The figure (Fig. 1) shows the membership function for a fuzzy number "approximately x_m".

In this study, a multi-period newsvendor problem with pre-season extension under fuzzy demand is analyzed. As the selling season draws closer, suppliers lead times shortens and the production cost increases. In contrast, demand fuzziness decreases and more accurate demand forecasts can be maintained that lead to lower overage/underage costs.

The purpose of the model is to find the best order period (M*) and best order quantity (Q*)for the multi-period newsvendor problem with pre-season extension under discrete fuzzy demand $\widetilde{X}$ with membership function $\mu_{\widetilde{X}}(x_i)$, precise production cost c_p, fuzzy unit holding cost $\widetilde{c_h}$ with $\mu_{\widetilde{c_h}}$ and fuzzy unit shortage $\widetilde{c_s}$ cost with $\mu_{\widetilde{c_s}}$.

3.1. Fuzzy demand and fuzzy inventory costs with precise purchasing cost

Let the membership function $\mu_{\widetilde{X}}(x_i)$ of fuzzy demand $\widetilde{X}$ given by domain $\widetilde{X}=\{x_0, x_1, x_2, ..., x_n\}$ have the triangular membership function (Fig. 1). Unit production cost, (c_p) is precise while unit holding cost ($\widetilde{c_h}$) and unit shortage cost ($\widetilde{c_s}$), are represented by triangular fuzzy numbers. The uncertain demand and uncertain cost parameters cause uncertain overage and underage costs. For a given Q and $x_i \in \widetilde{X}$, the fuzzy overage $(\widetilde{OC})$ and underage costs $(\widetilde{UC})$, are as follows;

$$\begin{aligned}\widetilde{OC} &= \widetilde{c}_h \max\left\{\left(Q - \widetilde{X}\right), 0\right\} \\ \widetilde{UC} &= \widetilde{c}_s \max\left\{\left(\widetilde{X} - Q\right), 0\right\}.\end{aligned} \tag{12}$$

The unit penalty cost $(\widetilde{PC})$ is the sum of unit overage cost and unit underage cost with the membership function $\mu_{\widetilde{PC}}$.

$$\widetilde{PC} = \widetilde{OC} + \widetilde{UC}. \tag{13}$$

The unit penalty cost $(\widetilde{PC}))$ is a level-2 fuzzy set which means that it includes two fuzzy values and there are corresponding membership degrees of these fuzzy values.

A level-2 fuzzy set can be reduced to an ordinary fuzzy set by s-fuzzification process (Zadeh 1971). The membership function of an ordinary fuzzy set is maintained via s-fuzzification (“s” stands for support) as follows:

$$\mu_{s-fuzz(\widetilde{PC})}(x) = \sup_{i=0,1,2,\ldots,n}[\mu_{\widetilde{PC}}(i) \times \mu_{c_i}(x)], x \in \widetilde{X}, \tag{14}$$

where $c_i(x)$ is the i^{th} possible fuzzy cost of $\widetilde{PC}$ and $\mu_{\widetilde{PC}}(i)$ is the possibility of that cost.According to the properties of possibility measure (Zadeh 1978), $\mu_{\widetilde{PC}}(i)$ is obtained as follows:

$$\mu_{\widetilde{PC}}(i) = \max_{x_i \in \widetilde{X}} \mu_{\widetilde{X}}(x_i), \quad i = 0,1,2,\ldots,n. \tag{15}$$

The fuzzy total cost is minimized by the best order quantity (Q*):

$$\widetilde{TC}(Q^*, \widetilde{X}) = \min_Q \left[c_p Q + defuzz(s - fuzz(\widetilde{PC}))\right]. \tag{16}$$

Here, the operator “defuzz” denotes the defuzzification process. Defuzzification is the conversion of a fuzzy quantity to a precise quantity; in contrast fuzzification is the conversion of a precise quantity to a fuzzy quantity. Usually, a fuzzy system will have a number of rules that transform a number of variables into a “fuzzy” result. Defuzzification would transform this result into a single number (Ross 1995). There are several methods for defuzzification. In this study centroid defuzzification method is applied. Centroid defuzzification method (also called center of area, center of gravity) is the most common of all the defuzzification methods. It is given by the algebraic expression as follows.

For continuous functions:

$$z^* = \frac{\int \mu_{\widetilde{C}(z)} z dz}{\int \mu_{\widetilde{C}(z)} dz}. \tag{17}$$

For discrete functions:

$$z^* = \frac{\sum \mu_{\widetilde{C}(z)} z}{\sum \mu_{\widetilde{C}(z)}}, \tag{18}$$

where $\widetilde{C}_{(z)} = \bigcup_{i=1}^{z} \widetilde{C}_i$ and $\widetilde{C}_i$ is one of the membership functions those figure the fuzzy output. This method is represented in Fig. 2.

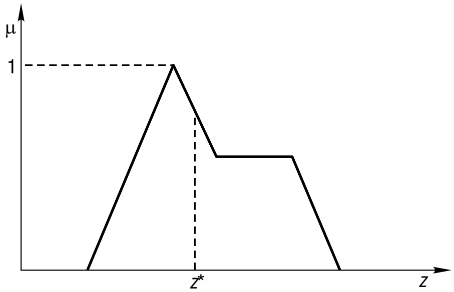

Fig. 2. Centroid defuzzification method

Best order quantity (Q*) which minimizes the fuzzy total cost is found by brute-force search algorithm which is a very general problem-solving technique that consists of systematically enumerating all possible candidates for the solution and checking whether each candidate satisfies the problem's statement.

The procedure explained above is examined for every pre-season ordering period. Through these periods as the selling season draws closer the production cost increases and demand fuzziness decreases and more accurate demand forecasts can be maintained that lead to lower overage/underage costs.

In the multi-period model, c_{p_j} denotes unit production cost for period j. Demand fuzziness decreases as the ordering period closes to the selling season. This decrease of fuzziness is maintained by fuzzy concentration of the membership function of demand according to the periods. The membership function $\mu_{\widetilde{X}}(x_i)$ of fuzzy demand $\widetilde{X}$ is given by domain $\widetilde{X} = \{x_0, x_1, x_2, ..., x_n\}$ concentrates as the selling season draws closer.

Concentrations tend to concentrate the element of a fuzzy set by reducing the degree of membership of all elements that have lower membership degrees in the set. The less an element is in a set, the more it is reduced in membership through concentration (Ross 1995).

The membership function of fuzzy demand for period j is denoted by $\mu_{\widetilde{X}_j}(x_i)$. Fuzzy total cost for pre-season ordering periods is as follows:

$$\widetilde{TC}_j(Q_j^*, \widetilde{X}) = \min_{Q_j}[c_{p_j} Q_j + defuzz(s - fuzz(\widetilde{PC}_j))]. \tag{19}$$

Here $\widetilde{PC}_j$ represents the penalty cost for period j where $j=1,2,...,m$. The best order period (M^*) is the period which has the minimum fuzzy total cost.

3.2. Numerical illustration

Suppose that a retailer wants to introduce an innovative product to the market at the beginning of July and have six possible periods (January, February, March, April, May, June) to give the manufacturing order to the manufacturer. As the selling season draws closer, suppliers lead times shorten and the production cost increases. On the other hand demand fuzziness decreases and more accurate demand forecasts can be maintained that lead to lower overage/underage costs.

Let unit holding cost $(\widetilde{c}_h)$ and unit shortage cost $(\widetilde{c}_s)$ be imprecise and represented with triangular fuzzy numbers as, $\widetilde{c}_h = \$[1,2,3]$ and $\widetilde{c}_s = \$[4,5,6]$, respectively. Unit purchasing cost is precise and increases 5% according to the previous months' cost Table 1.

Table 1. Unit purchasing cost per month ($)

Jan	Feb	March	April	May	June
3.50	3.68	3.86	4.05	4.25	4.47

Let the membership function $\mu_{\widetilde{X}}(x_i)$ of fuzzy demand $\widetilde{X}$ for January be $\widetilde{X} = \{1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000, 5500, 6000\}$ with the triangular membership function as follows:

$$\mu_{\widetilde{X}}(x_i) = \begin{cases} 0 & x_i < 1000 \\ \left(\dfrac{x_i - 1000}{2500}\right) & 1000 < x_i < 3500 \\ \left(\dfrac{6000 - x_i}{2500}\right) & 3500 < x_i < 6000 \\ 0 & x_i > 6000 \end{cases} \quad (20)$$

For example let x_1 = 1000. The membership value of x_1 will be $\mu_{\widetilde{X}}(x_1) = 0$ or if x_7 = 1000 then the membership value of x_7 will be $\mu_{\widetilde{X}}(x_7) = 0.80$ and so on. The membership functions for other months are maintained through the membership function of the demand for January using fuzzy concentration (Table 2 and Fig. 3).

Table 2. Membership values of demand per month

	Jan	Feb	March	April	May	June
Demand	$\mu_{\widetilde{X}}(x_i)$	$\mu_{\widetilde{X}}(x_i)^{1.25}$	$\mu_{\widetilde{X}}(x_i)^{1.5}$	$\mu_{\widetilde{X}}(x_i)^{2}$	$\mu_{\widetilde{X}}(x_i)^{3}$	$\mu_{\widetilde{X}}(x_i)^{4}$
1000	0.000	0.000	0.000	0.000	0.000	0.000
1500	0.200	0.134	0.089	0.040	0.008	0.002
2000	0.400	0.318	0.253	0.160	0.064	0.026
2500	0.600	0.528	0.465	0.360	0.216	0.130
3000	0.800	0.757	0.716	0.640	0.512	0.410
3500	1.000	1.000	1.000	1.000	1.000	1.000
4000	0.800	0.757	0.716	0.640	0.512	0.410
4500	0.600	0.528	0.465	0.360	0.216	0.130
5000	0.400	0.318	0.253	0.160	0.064	0.026
5500	0.200	0.134	0.089	0.040	0.008	0.002
6000	0.000	0.000	0.000	0.000	0.000	0.000

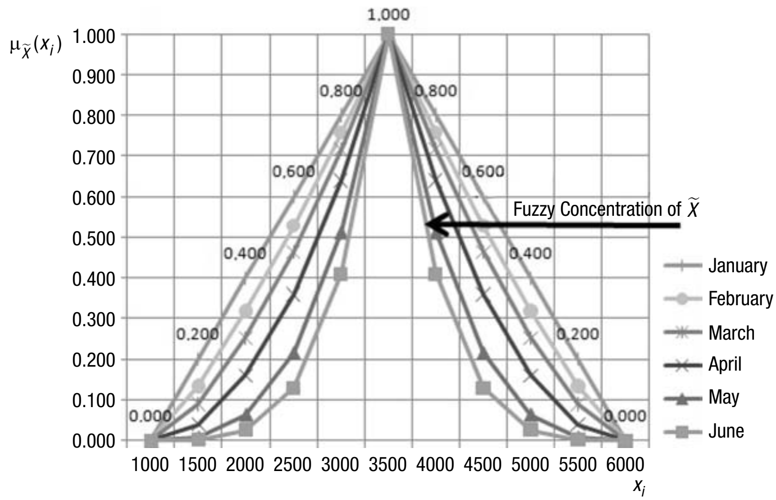

Fig. 3. Fuzzy concentration of $\widetilde{X}$

For example, let us order a quantity of 2000 units at the beginning of January. The unit penalty cost $(\widetilde{PC})$ is a level-2 fuzzy set including imprecise demand and costs. For x_1 = 1000, the fuzzy penalty cost will be $\widetilde{PC} = (1,2,3) \times 1000 = (1000, 2000, 3000)$ with possibility of 0. For x_5 = 3000, the fuzzy penalty cost will be $\widetilde{PC} = (4,5,6) \times 1000 = (4000, 5000, 6000)$ with the possibility 0.80 and so on. For the ordering period January fuzzy unit penalty cost values for Q = 2000 is given in Table 3.

The graphical representations of level-2 fuzzy sets of $(\widetilde{PC})$ for January when Q = 2000 and the corresponding s-fuzzified set $s - fuzz(\widetilde{PC})$ are shown in Figs. 4a and 4b, respectively.

Table 3. Unit penalty cost values for January, Q = 2000

	$\widetilde{X}$	$\mu_{\widetilde{X}}(x_i)$	$(\widetilde{OC})$	$(\widetilde{UC})$	$(\widetilde{PC})$
x_1	1000	0	(1000,2000,3000)	(0,0,0)	(1000,2000,3000)
x_2	1500	0.20	(500,1000,1500)	(0,0,0)	(500,1000,1500)
x_3	2000	0.40	(0,0,0)	(0,0,0)	(0,0,0)
x_4	2500	0.60	(0,0,0)	(2000,2500,3000)	(2000,2500,3000)
x_5	3000	0.80	(0,0,0)	(4000,5000,6000)	(4000,5000,6000)
x_6	3500	1	(0,0,0)	(6000,7500,9000)	(6000,7500,9000)
x_7	4000	0.80	(0,0,0)	(8000,10000,12000)	(8000,10000,12000)
x_8	4500	0.60	(0,0,0)	(10000,12500,15000)	(10000,12500,15000)
x_9	5000	0.40	(0,0,0)	(12000,15000,18000)	(12000,15000,18000)
x_{10}	5500	0.20	(0,0,0)	(14000,17500,21000)	(14000,17500,21000)
x_{11}	6000	0	(0,0,0)	(16000,20000,24000)	(16000,20000,24000)

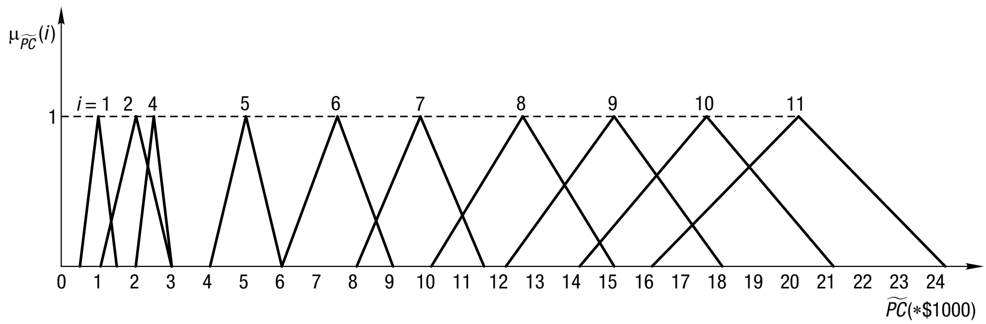

Fig. 4a. Level-2 fuzzy set $(\widetilde{PC})$

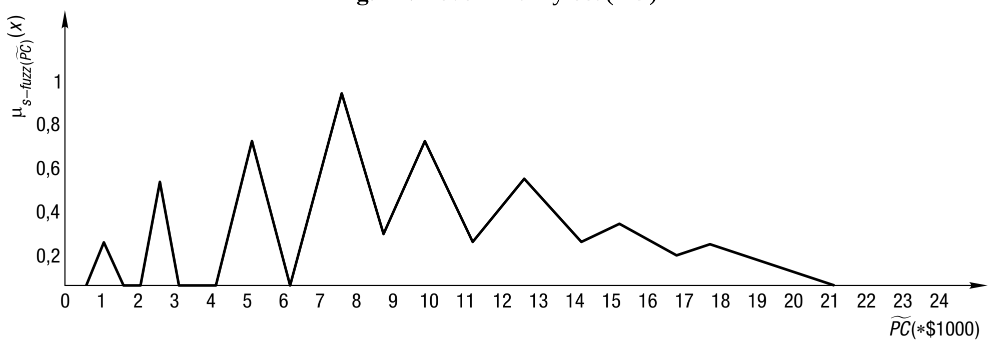

Fig. 4b. $s-fuzz(\widetilde{PC})$

According to the s-fuzzified value of the penalty cost, total cost for the given values when Q = 2000 is calculated as below:

$$\widetilde{TC}(2000,\widetilde{X}) = [3.50 \times 2000 + defuzz(s - fuzz(\widetilde{PC}))]. \tag{21}$$

Here, the operator "defuzz" denotes the centroid method for defuzzification. Centroid defuzzification values have been obtained by using MATLAB R2008a Fuzzy Logic Toolbox as in Fig. 5.

$$\widetilde{TC}(2000,\widetilde{X}) = [3.5 \times 2000 + 10,022] = \$17,022. \tag{22}$$

Fuzzy total cost values for all other order quantities of the ordering period January are given in Table 4. For the ordering period January, best order quantity which gives the min fuzzy total cost is found 3000 units with the fuzzy total cost value $16,820.

The procedure continues with calculation of other periods best order quantities. Using Equation 19, the fuzzy total costs for pre-season ordering periods are calculated and given as in Table 5 and Fig. 6.

As stated by Table 5, best order period (M*) is found January with fuzzy total cost $16,820 with 3000 units. Here the decrease of demand fuzziness causes a decline on the fuzzy total cost values among ordering periods. However, the change of production cost also affects the cost function. The proposed model offers best order period and corresponding order quantity for the given fuzzy variables.

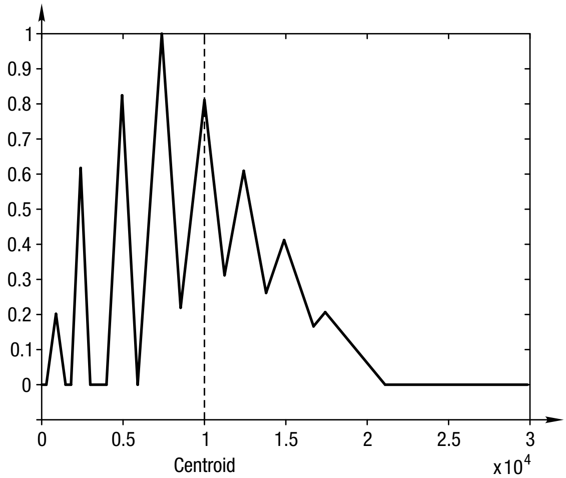

Fig. 5. Centroid defuzzification of $s-fuzz(\widetilde{PC})$

Table 4. Fuzzy total costs for January, ($)

Order quantity	Fuzzy total cost
1000	17,356
1500	17,159
2000	17,022
2500	16,930
3000	16,820*
3500	16,828
4000	17,258
4500	19,079
5000	21,514
5500	24,045
6000	26,681

Table 5. Fuzzy total cost for pre-season ordering periods, $\widetilde{c_h}=\$[1,2,3]$, $\widetilde{c_s}=\$[4,5,6]$

Q	$\widetilde{TC}(Jan)$	$\widetilde{TC}(Feb)$	$\widetilde{TC}(March)$	$\widetilde{TC}(April)$	$\widetilde{TC}(May)$	$\widetilde{TC}(June)$
c_p	3,50	3,68	3,86	4,05	4,25	4,47
1000	17,356	17,335	17,360	17,327	17,262	17,332
1500	17,159	17,161	17,219	17,184*	17,098	17,202
2000	17,022	17,089	17,210*	17,202	17,087*	17,202*
2500	16,930	17,082	17,282	17,333	17,223	17,350
3000	16,820*	17,075*	17,375	17,515	17,448	17,617
3500	16,828	17,168	17,553	17,787	17,815	18,087
4000	17,258	17,718	18,237	18,659	19,003	19,589
4500	19,079	19,709	20,401	21,063	21,708	22,508
5000	21,514	22,252	23,059	23,861	24,678	25,636
5500	24,045	24,902	25,831	26,775	27,756	28,843
6000	26,681	27,658	28,703	29,778	30,880	32,058

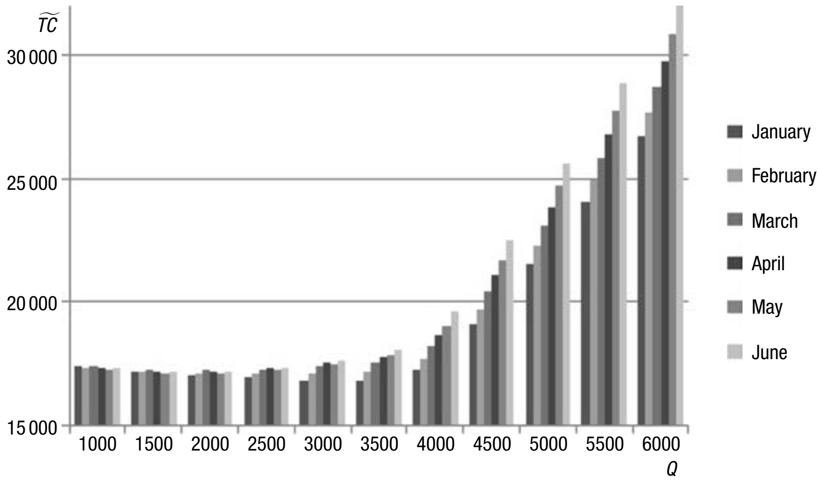

Fig. 6. Fuzzy total cost for pre-season ordering periods

3.3. Sensitivity analysis

In this section, various experiments have been performed to analyze the effect of membership function shapes to the fuzzy models. Through these experiments fuzzy unit holding cost and fuzzy unit shortage cost values have been changed (Table 6).

Table 6. Performed experiments and values of the cost parameters

Experiments	$\widetilde{c_h}$($)	$\widetilde{c_s}$($)
1	(1,2,3)	(4,5,6)
2	(1,2,3)	(4,5,8)
3	(1,2,3)	(4,5,10)
4	(1,2,3)	(4,5,12)
5	(1,2,5)	(4,5,6)
6	(1,2,5)	(4,5,8)
7	(1,2,5)	(4,5,10)
8	(1,2,5)	(4,5,12)
9	(1,2,7)	(4,5,6)
10	(1,2,7)	(4,5,8)
11	(1,2,7)	(4,5,10)
12	(1,2,7)	(4,5,12)
13	(1,2,9)	(4,5,6)
14	(1,2,9)	(4,5,8)
15	(1,2,9)	(4,5,10)
16	(1,2,9)	(4,5,12)

In these experiments, production cost values and demand membership functions are considered the same as the previous illustration while the membership of holding cost and the membership of shortage cost have been changed. According to the provided membership values, the fuzzy model generates the results represented in Table 7.

Table 7. M^*, Q^* and $\widetilde{TC}$ for different $\widetilde{c_h}$ and $\widetilde{c_s}$

Experiments	M^*	Q^*	$\widetilde{TC}$($)
1	Jan	3000	16,820
2	Jan	2500	17,716
3	May	2500	18,484
4	June	3000	19,152
5	Jan	3000	16,626
6	Jan	3000	17,619
7	May	2500	18,457
8	May	3000	19,093
9	Jan	3000	16,509
10	Jan	3000	17,545
11	May	3000	18,431
12	May	3000	19,062
13	Jan	3000	16,449
14	Jan	3000	17,498
15	May	3000	18,417
16	May	3000	19,048

The results provided in Table 7 represents that, the increase of the fuzziness of cost parameters causes an increase of fuzzy total cost values. For example, in experiment 2, we increased the fuzziness of shortage cost by changing the membership function shape to $\widetilde{c_s} = \$[4,5,8]$. This change increased the fuzzy total cost value from $16,820 to $17,716. Shortage cost values as they have higher fuzziness affect the order period in that way. In experiments 3 and 4 membership values of shortage costs are wider than the values in experiments 1 and 2. For the experiments 1 and 2 best order period is found as January which has the maximum demand fuzziness and minimum production cost. However, for experiments 3 and 4 best order period changes to May and June, respectively, which have minimum demand fuzziness and maximum production costs.

Best order quantities for the experiments do not change much according to different scenarios. The reason of this situation is the symmetrical shape of demands' membership function. Only in three experiments best order quantity decreases from 3000 to 2500. If nonsymmetrical membership functions are used for demand data, more changes on offered order quantities can be observed.

4. Conclusion

The classical single-period inventory problem has been considered extensively in the relevant literature. Through these studies, most of the extensions have been made in the probabilistic and possibilistic framework. Most of the papers under possibilistic framework proposed single-period inventory models with only fuzzy environment extensions. Through this paper, we combine two extensions to single-period inventory problem which are; an extension to fuzzy environment and an extension to models with more than one period to prepare for the selling season. The model is developed for innovative products. The demand, holding and shortage cost parameters are represented by fuzzy numbers. The model has been experimented with an illustrative example and supported by sensitivity analyses.

Contrary to the crisp model, fuzzy model proposes highly flexible solutions for all possible states. When the solutions of the model are analyzed, it is seen that the decrease of demand fuzziness causes a decline on the fuzzy total cost values among ordering periods. The fuzzy model offers the best order period and corresponding order quantity for the given fuzzy variables. Furthermore, the increase of the fuzziness of cost parameters causes an increase in fuzzy total cost values. Shortage cost values as they have higher fuzziness affect the order period in that way. Best order quantities for the experiments do not change much according to different scenarios. The reason of this situation is the symmetrical shape of demands' membership function.

For further research, we suggest the examination of the influence of nonsymmetrical and different type of membership functions for demand data on order quantities. Furthermore, the use of an imprecise continuous demand function instead of the discrete case of this paper can be examined. This will require optimization techniques for solution procedure. Lastly, dependent demand structure among ordering periods is suggested to be applied.

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KELIŲ LAIKOTARPIŲ PARDAVIMŲ MODELIS, PAPILDYTAS PARUOŠIAMUOJU LAIKOTARPIU, ESANT NERAIŠKIAJAI PAKLAUSAI

H. Behret, C. Kahraman

Santrauka

Straipsnyje pasiūlytas neraiškusis kelių laikotarpių pardavimo modelis, papildytas paruošiamuoju laikotarpiu inovatyviems produktams. Produkto paklausa apibūdinama neraiškiaisiais skaičiais, aprašytais trikampe priklausomumo funkcija. Turto ir sąnaudų parametrai laikomi netiksliais ir taip pat apibūdinami trikampe priklausomumo funkcija aprašytais neraiškiaisiais skaičiais. Kai pardavimo laikotarpis priartėja, tiekimo laikas trumpėja, o gaminio kaina išauga. Priešingai, dėl praeinančio pardavimo laikotarpio paklausos neapibrėžtumas sumažėja ir galima tiksliau prognozuoti paklausą, o tai padeda sumažinti išlaidas dėl gaminių pertekliaus ar trūkumo. Šio modelio tikslas – rasti geriausią užsakymo laiką ir geriausią užsakymo kiekį, kurie padėtų sumažinti visų numatomų sąnaudų neapibrėžtumą. Pateikiamas modelio taikymo pavyzdys ir modelio jautrumo analizė.

Reikšminiai žodžiai: atsargų problema, neraiškusis modeliavimas, pardavimas, inovatyvus produktas, neraiškioji paklausa, neraiškioji atsargų savikaina, paruošiamasis laikotarpis.

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