

COALITION OR DECENTRALIZATION: A GAME-THEORETIC ANALYSIS OF A THREE-ECHELON SUPPLY CHAIN NETWORK

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Abstract. Supply chains have become the major and dominant paradigm of business and competition. The main challenge is how to act in multi-echelon supply chains considering the levels involved. Making a choice independently or integrating with some or all levels will be a critical decision, and therefore affects the overall profit of the chain. This article proposes a non-cooperative game theory approach to helping in making a better decision in the supply chain and gaining the most accessible benefit. Our research considers unlimited three-echelon supply chains with S suppliers, M manufacturers and K retailers. The Nash equilibrium and definition are used bearing in mind inventory and pricing and marketing cost as decision variables for this matter. This paper studies a three-echelon supply chain network and focuses on the value of integrating a pair of partners in the chain. In the decentralized case, the supplier sets its own price, the manufacturer points out order quantity, wholesale price and backorder quantity, and the retailer charges the final retail price of the product and marketing product. Though there are multiple players at a single echelon level, each manufacturer supplies only a specific product to a given retailer. In addition to the decentralized case, two integration scenarios have been taken into account: manufacturer-retailer and supplier-manufacturer. As for manufacturer-retailer integration, inventory/holding cost issues diminish to a single warehouse and the retailer does not have to enforce marketing effort any more. Supplier-manufacturer integration brings similar benefits. Under each scenario, all parties involved simultaneously set their strategies. Through a numerical experiment, 17 design cases (through designing experiments) have been developed and the total profit of the supply chain under each scenario has been evaluated. Statistical tests on the above introduced 17 experiments have found that the decentralized system performs significantly worse than the integration of the supplier with the manufacturer, whereas no significant difference can be observed regarding other combinations.

Keywords: supply chain, coalition, non-cooperative games, Nash equilibrium, design of experiment, Levenberg-Marquardt algorithm.

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Introduction

More than 50 years ago, Forrester (1958) introduced the elements of a theory that today is called supply chain management (SCM). The concept of the supply chain means that many experts believe that competition is transferred from companies to chains. SCM is extensive enough, and therefore large international corporations such as Cisco, Dell Computer, Gillette, Kodak, LEGO, Motorola, Sony, 3M, Xerox and Wal-Mart implemented it in the past decade. Moreover, international consultancy firms like IDM business Consulting Services, A.T. Kearney, Cap Gemini, etc. have adopted SCM as an important business area. Also, a large number of universities and business schools have included SCM courses in their curricula. Many scholars and experts gave different definitions for SCM that depend on their viewpoints and attitudes (Walker 2005). The role and importance of supply chain management have faced a number of challenges and problems. Although a comprehensive model dealing with the issues of the supply chain has not been explained, we have to indicate that questions such as reviewing the theoretical foundations of information systems, marketing, financial management, logistical and organizational relations have been considered by many researchers (Wang *et al.* 2007). There are many challenges that latent in the concept of the supply chain. The decisions made in SCM are mainly about the flows between chain stages. Therefore, many scholars express challenges and problems, and SCM have tried to answer them (Chandra, Kamrani 2004; Chopra, Miendel 2007; Simchi-Levi *et al.* 2004; Wisner *et al.* 2008). The objective of supply chain management is to improve various activities and components to increase the overall benefits of the supply chain system. Many decisions are made at a different level of the supply chain, which includes detailed and strategic decisions. Planning important decisions in a multi-echelon supply chain will affect all levels and the SC as a whole (Stadtler, Kilger 2007). If each level of the supply chain makes their inventory, the decision on pricing and advertising without considering other levels as well as the bullwhip effect will occur and the advantage of supply chain competitiveness decrease (Lee *et al.* 1997). Between the components and different levels of the supply chain, in order to achieve the overall objectives, many contradictions may occur, including that these disorders, over time, may result in the decreased strength and competitiveness of the supply chain. Such conflicts, like marketing costs (advertising), pricing and inventory decisions can occur during the life cycle of the supply chain. For avoiding such loss in the SC, many coordination mechanisms have been introduced in recent researches. There are many possible interactive coordination mechanisms that can occur between different levels (Esmaeili *et al.* 2008). A large part of these mechanisms are based on a game theory approach. The game theory is concerned with the actions of decision makers who are conscious that their actions affect each other. The game theory approach is an appropriate tool for collaboration in the supply chain.

Beside contradiction with decision variables, different levels of the supply chain may decide on acting independently or, in some cases, trying to integrate with some other levels for gaining more advantages. In this case, the main challenge is how to act in multi-echelon supply chains taking into account the levels involved. Making a choice independently or integrating with some or all levels will be a critical decision, and therefore affects the overall profit of the chain. This decision will have an influence on prices, inventory, lot sizes and costs which will finally change the overall profit of the supply chain. For solving this problem and finding a suitable answer, a three-echelon unlimited supply chain with S suppliers, M manufacturers and K retailers has been considered. In addition, decisions on pricing, inventory and advertising are included as three main decision variables in the proposed models. By using the definition of the Nash equilibrium for continuous problems, the best responses for each level of the supply chain in decentralization and integration situations are identified and used in a simulated supply chain. By comparing the obtained results, the best decisions are illustrated. The remainder of this paper is organized as follows. First, supply chain management, the game theory and the Nash equilibrium are introduced and the classification of researches on similar topics is illustrated. Next, the assumptions and notations of our proposed models are presented and the payoff functions of each player in all situations (integration or decentralization) are designed. After, the best response of each player is calculated. Finally, by using the proposed models in the simulated SC, the results are compared and the final conclusion is proposed.

1. Literature review

This section includes the basics and concepts of the game theory, different types of coordination contracts and reviews similar researches. As a definition, the supply chain consists of all parties involved, directly, or indirectly, in fulfilling a customer request (Chopra, Meindel 2007) and mentioning all activities performed until a raw material is delivered as the final good to a customer (Gumus, Guneri 2007). These activities and functions include new product development, marketing, operations, distributions, financing and customer services. A typical supply chain may involve a variety of stages such as customers, retailers, wholesalers, distributors, manufacturers and raw material suppliers (Chopra, Meindel 2007). Between the components and different levels of the supply chain, in order to achieve the overall objectives, many contradictions may occur, the contradictions that these disorders take place over time result in the decreased strength and competitiveness of the supply chain. One of the main tools for solving the problem in the before mentioned situation are the game theory approach. The essential elements of the game are players, actions, payoffs and information (Chen 2009). These are collectively known as the rules of the game, and the objective of the modeller is to describe the situation in terms of the rules of the game so as to explain what will happen in that situation. Trying to maximize their payoffs, the players will devise plans known as strategies that pick actions depending on the information that has arrived at each moment. The combination of strategies chosen by each player is known as equilibrium while given which modeller can see what actions come out of the conjunction of all players' plans, which tells him the outcome of the game.

While information transaction is not possible between different players (different layers of the SC), in such situations and by considering the Nash definition, each player will stimulate competitor believes or best responses, and, when these believes are correct, the Nash equilibrium will occur (Osborne 2004). In the given two-player game, the best responses are defined as (1):

$$\begin{aligned}
 B_1(S_2^*) &= \{S_1^* : U_1(S_1^*, S_2^*) > U_1(S_1, S_2^*) ; \forall s_1 \in S_i\}, \\
 B_2(S_1^*) &= \{S_2^* : U_2(S_1^*, S_2^*) > U_2(S_1^*, S_2) ; \forall s_2 \in S_i\}, \\
 B_i(S_{-i}) &= \{S_i : U_i(S_i, S_{-i}) > U_i(S_i', S_{-i}) ; \forall s_i \in S_i\},
 \end{aligned}
 \tag{1}$$

where B_i stands for the best response for player i , (S_i, S_{-i}) – for the strategy chosen by player i & $-i$, $(i, -i)$ – for two players of the game, (S_i^*, S_{-i}^*) – for the best strategies for players, $U_i(S_i, S_{-i})$ – for utility or payoff when players choose (S_i, S_{-i}) as their final decisions. By considering the definition of the best response and continuous and discrete payoff functions, the Nash equilibrium is computable by the derivation of utility or the payoff function of each player regarding a specific decision variable. The Nash game, definition and equilibrium are used in famous situations such as the Bertrand duopoly model, Cornet model of duopoly, the final offer arbitration and the problem of commons (Gibbons 2002).

Game theoretic analysis for supply chain networks includes a wide range of research. Some scientists have focused on the use of the Nash equilibrium in supply chain coordination by applying a profit sharing contract (Feng *et al.* 2007; Jiazhen, Qin 2008; Feng 2008; Ying *et al.* 2007; Jaber *et al.* 2006; Bai, Wang 2008; Xu, Zhong 2011; Liu, Zhang 2006; Wang *et al.* 2009). The other contracts of supply chain coordination are presented in Table 1 (Govindan, Nicoleta 2011).

Besides using coordination contracts, Nash and Stackelberg games also can be used for coordinating the SC in complete situations with a determined demand function (Leng, Parlar 2010; Arda, Hennem 2005). Besides, Zandi *et al.* (2012) proposed a strategic theoretic approach of a cooperative game to market segmentation. For a symmetric supply chain, some researchers gave closed-form expressions of unique equilibrium (Adida, Demiguel 2011). In addition, equilibrium for the Shapley value and Eliasberg model for coordination and cooperation problems in the SC are performable (Zhao *et al.* 2010; Leng, Zhu 2009). In some cases, while incomplete information on decisions and payoffs dominate game conditions, Bayesian – Nash Games are used (Wang, Zhao 2009; Cachon, Lariviere 1999). Other optimization tools such as the queuing theory, a Markov chain, backward induction, stochastic programming and a genetic algorithm for solving coordination and cooperation problems in the supply chain, mostly in the situations of incomplete information games, are also performable (Cachon, Kok 2010; Hennem, Arda 2008; Stein, Ginevicius 2010; Kaviani *et al.* 2011; Gupta, Weerawat 2006).

Table 1. Coordination contracts

Contract	Description
Wholesale	The buyer pays a fixed and quantity-independent price of each purchased unit for the seller
Discount	Quantity-dependent unit prices
Buyback	The seller promises to compensate the buyer for unsold quantities
Revenue sharing	The downstream agent commits to return a pre negotiated portion of its realized profits to the upstream agent
Rebate	The upstream agent rewards the downstream agent for every sold unit
Side payment	Lump-sum monetary transfers among the contracting agents that are independent of the amount of trade and used as compensation and incentive alignment mechanisms
Flexible	In contrast with a rebate contract
Push & pull	By increasing the amount of selling or buying, the upstream agent proposes a lower price

Considering uncertainty in the game theory approach to solving different dilemmas is really noticeable, which eventuates to multi-criteria decision making (MCDM) or multi-objective decision making (MODM) usage in the game theory. Therefore, in 2005, Peldschus and Zavadskas proposed a Fuzzy matrix game by multi-criteria modelling of decision making in engineering projects (Peldschus, Zavadskas 2005). Three years later, a new logarithmic normalization method in the game theory was suggested (Zavadskas, Turskis 2008). Same time after, an overview of MCDM methods and its application to economics based on the game theory was figured by the same authors (Zavadskas, Turskis 2011). Recent findings have illustrated the application of the MODM and game theory approach where Peldschus and Zavadskas proposed an equilibrium approach to construction processes by multi-objective decision making for construction projects (Peldschus, Zavadskas 2012). As some of the aforementioned researches indicate, in practice, the application of the game theory in the fields of engineering and construction has further developed (Zavadskas *et al.* 2004; Peldschus 2008; Kaplinski, Tamosaitiene 2010; Peldschus *et al.* 2010).

This research mainly considers the use and application of the game theory, especially non-cooperative Nash games, in supply chain management. In Nash games, all levels of the SC or each player in the game act simultaneously with complete information about the game, players and equal power. Most of the games used in the SC are non-repetitive, with two or finally three levels and limited to one or finally two members at each level. Inventory, pricing and marketing policies are included in researches. Our paper reflects on inventory with shortage (backlog), incremental production, a nonlinear cost production function for the manufacturer, a nonlinear demand function, semi integrated (coalition) games and an unlimited supply chain, which differs this research from others.

2. Basic model

A three-echelon supply network made of K retailers, M manufacturers and S upstream suppliers where $K \geq 1, M \geq 1, S \geq 1$, (Fig. 1) has been considered. This network produces, distributes and sells multiple products to the end customers (i.e. consumers):

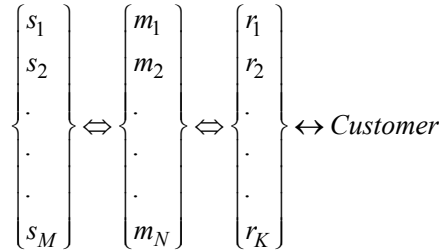


Fig. 1. The three-echelon supply network

In this supply network, we assume that all agents are able to decide independently or, in some cases, integrate with the agents of other levels of the network. The network follows a make-to-order pull system in which orders first pass from retailers to manufacturers, and then from manufacturers to suppliers. We assume that all agents have complete information. In addition, shortages (and hence stock out) are allowed for manufacturers. The cost of shortages will be considered for the manufacturer during a shortage period, but no shortage is assumed for retailers and suppliers. Consumer demand for the product depends on both the retail price and marketing cost used for product advertisement. Following literature (Lee 1993, Esmaeili *et al.* 2008; Jia *et al.* 2013), we acknowledge a deterministic non-linear form of a consumer demand function as given by (2):

$$D_n = k.P_r^{-\alpha}.C_{M_n}^\beta, \tag{2}$$

where D_n presents demand for product n , P_r denotes the selling price of product n by retailer γ , C_{M_n} points to marketing cost for product n , k, α, β , are all strictly positive constants to represent the corresponding coefficient in the demand function.

The total inventory cost of each manufacturer includes its shortage cost and inventory holding cost and it is computable by (3), in which C_{h_n} stands for holding cost estimated by the manufacturer, $(\lambda_n = 1 - \frac{D_n}{PC_n})$ denotes production rate, Q_r is production quantity, B_n represents manufacturer’s shortage, C_{B_n} is the cost of manufacturer’s shortage, PC_n is production capacity provided by the manufacturer (Jia *et al.* 2013):

$$[C_{h_n} \cdot \frac{(\lambda_n \cdot Q_r - B_n)^2}{2 \cdot \lambda_n \cdot Q_r}] + [\frac{C_{B_n} \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_r}]. \tag{3}$$

Notice that the model presented in (3) follows literature (Oganezov 2006; Wang, Tang 2009; Chakraborty *et al.* 2010; Chang 2008; Pentico *et al.* 2009; Jia *et al.* 2013).

As a remark, the final retail selling price and retail marketing cost, production quantity and shortage quantity, besides the mass production price for the manufacturer, and finally the raw material price for suppliers are the main decision variables of this research. The cost of unit production is nonlinear function $C_{P_n} = u.D_n^{-\gamma}$ decreasing in demand and having coefficients $u > 0, \gamma > 0$ (Bazaraa *et al.* 1993). Regarding this supply network, the scenario when each manufacturer supplies only a specific product to a specific retailer $n = r = m$ is considered. However, upstream suppliers sell their raw materials to any manufacturer when needed. This matches the scenario in many industries such as fashion apparel in which many retailers (fashion retail brands) have designed garment factories (manufacturers) to produce solely for them while since there are relatively few upstream suppliers (for fabrics or components such as zippers) in the market, most manufacturers will source from the well-established ones (e.g. YKK is the zipper supplier to all kinds of manufacturers and all kinds of retail brands). Following the standard norm discussed in literature, we assume, throughout this paper, that each player at any level of the supply network is fully rational. To enhance the presentation, the notation employed in Table 2 of this paper is summarized.

Table 2. The notation of decision variables and parameters

Description	Note	Description	Note
Product n and ordering cost of supplier s	$C_{O_{sn}}$	Retailer's margin	G_r
Manufacturer's margin	G_n	Selling price of product n to retailer r	P_n
Manufacturer's total revenue	TR_n	Retailer's setup cost	$C_{s_{rn}}$
Total manufacturing cost	TC_n	Retailer's holding cost coefficient	k'_n
Supplier's margin	G_S	Retailer's total revenue	TR_r
Supplier's total revenue	TR_S	Retailer's total cost	TC_r
Supplier's total cost	TC_S	Retailer's total payoff	Z_r
Supplier's unit cost of a raw material	G_S	Coefficient of a raw material in product n	k_{s_n}
Supplier's holding cost coefficient	k_{S_s}	Price of a raw material from s of product n	C_{P_s}
Supplier's ordering cost	C_{S_o}	Variable manufacturing cost	C_{S_n}
Integrated payoff function of the manufacturer and retailer			Z_{MR}
Integrated payoff function of the manufacturer and supplier			Z_{SM}

3. Modelling/payoff functions

This section presents the analytical payoff function for each agent of the supply network. Two different approaches are considered. First, trio players, including suppliers, manufacturers and retailers, decide independently without any coordination approach. Second, two parts of the SC merge each other and act as one player to confront the demonstration of the third party. In both approaches, mathematical models are illustrated and the best responses of each player are calculated based on the Nash equilibrium.

3.1. The payoff function and model of the retailer

For retailer r , it confronts holding and setup costs as well as purchasing cost from getting supply from the manufacturer. In addition, any retailer should have a positive margin to participate in the supply network. The income of each retailer involves revenue achieved by selling the final goods to the final customer. Considering the details above, the payoff function of the retailer and a related optimization model are shown in (4) (Jaafarnejad *et al.* 2012; Jia *et al.* 2013):

$$\begin{aligned}
 \text{Max } Z_r &= (k \cdot P_r^{-\alpha} \cdot C_{M_n}^\beta [P_r - P_n - C_{M_n} - C_{S_r} \cdot Q_r^{-1}]) - \left(\frac{1}{2} \cdot Q_r \cdot k_n' \cdot P_n\right) \\
 \text{s.t.} & \\
 P_r - P_n &\geq 0 \\
 D_n = k \cdot P_r^{-\alpha} \cdot C_{M_n}^\beta &\geq 0 \\
 D_n &\leq PC_n \\
 k > 0, \alpha > 1, 0 < \beta < 1, \alpha - \beta > 1, &
 \end{aligned}
 \tag{4}$$

where the first constraint implies that the final retail selling price is greater than the wholesale price paid to the manufacturer, and the second and third constraints guarantee that demand should not be negative or greater than production capacity. Decision variables associated with (4) in the supply network include P_r, C_{M_n}, P_n, Q_r .

3.2. The payoff function and model of the manufacturer

Manufacturer n confronts holding, setup, ordering and shortage costs, purchasing cost and production cost. The manufacturer receives revenue with the wholesale price. The payoff function of manufacturers and model constraints are shown in (5) where the first constraint implies that the wholesale price offered by the manufacturer to the retailer is greater than that paid for raw materials and where the second and third constraints ensure that demand should not be negative or greater than the available production capacity (Jia *et al.* 2013).

$$\begin{aligned}
 \text{Max } Z_n &= [(P_n - \sum_{s=1}^M (k_{s_n} \cdot C_{p_s}) - u \cdot D_n^{-\gamma}) \cdot D_n] - [(\sum_{s=1}^m (C_{O_{s_n}}) + \\
 &C_{S_n}) \cdot \frac{D_n}{Q_{r_n}}] - [C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n}}] - [\frac{C_{B_n} \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n}}] \\
 \text{s.t. :} \\
 P_n - [\sum_{s=1}^m (C_{P_s} \cdot k_{s_n}) + u \cdot D_n^{-\gamma}] &\geq 0 \\
 CP_n &\geq D_n \\
 D_n &= k \cdot P_n^{-\alpha} \cdot C_{M_n}^\beta \\
 k > 0, u > 0, \alpha > 1, 0 < \beta < 1, 0 < \gamma < 1, \alpha - \beta > 1.
 \end{aligned} \tag{5}$$

3.3. The payoff function and model of the supplier

A supplier has to bear holding, setup and purchasing costs. In return, every supplier will gain revenue by selling raw materials to manufacturers, and the total revenue depends on the amount of respective production used by the manufacturer. By considering the above indicated points, the payoff function of the supplier and its constraints are shown in (6), where the first constraint implies that the selling price of the raw material to the manufacturer should be greater than the procurement cost of acquiring the raw material by the supplier, and the second constraint guarantees that demand should not be negative (Jaafarnejad et al. 2012; Jia et al. 2013):

$$\begin{aligned}
 \text{Max } Z_S &= [(C_{P_s} - C_{S_o}) \cdot \sum_{n=1}^N k_{s_n} \cdot D_n] - [\sum_{n=1}^N \frac{D_n}{Q_{r_n}} \cdot C_{S_s}] - [\sum_{n=1}^N k_{S_s} \cdot C_{S_o} \cdot k_{S_n} \cdot \frac{Q_{r_n}}{2}] \\
 \text{s.t. :} \\
 C_{P_s} - C_{S_o} &\geq 0 \\
 D_n &= k \cdot P_n^{-\alpha} \cdot C_{M_n}^\beta > 0 \\
 k > 0, u > 0, \alpha > 1, 0 < \beta < 1, 0 < \gamma < 1, \alpha - \beta > 1.
 \end{aligned} \tag{6}$$

3.4. Channel integration

This section proposes that two levels of the SC make a joint venture to confront the leadership of the third party. We have considered two integration scenarios: manufacturer-retailer and supplier-manufacturer. In the first case, inventory/holding cost issues diminish to a single warehouse and the retailer does not have to enforce marketing effort any more. The second one brings similar benefits. Under each scenario, all parties involved simultaneously set their strategies.

3.4.1. Integration function (M-R Nash game) of the manufacturer – retailer

Our research is aimed at finding the best way of two-agent integration in the supply network to achieve the Nash equilibrium with the largest profit. As a remark, it is known that strategic partnership in a supply chain is most easily achievable and commonly seen in the case between two agents. Multi-agent integration (more than two) is relatively rare and usually done by involving a third part supply chain coordinator. In this paper, we confine ourselves to two-agent integration as commonly observed in the real world and assumed in the mainstream literature.

To achieve this goal, under our analysis, three options are possible for the three-echelon supply network: manufacturer- retailer integration (MR), supplier – manufacturer integration (SM) and supplier - retailer integration (SR). The purpose of this paper is to find the best integration mode among these three.

By vertically integrating manufacturers with retailers, the number of manufacturers is equal to the number of retailers, and the revenue of the MR pair will be the revenue generated by the retail selling price for the product sold. The incurred costs will include production costs, shortage costs, setup costs and holding costs. We have to notice that holding cost at this level includes one warehouse between manufacturers and retailers. In addition, marketing cost will not occur while manufacturers and retailers are bonded, because these two members are vertically integrated and simply controlled by one level in a centralized manner. A remark that marketing in our research is based on the costs the retailer spends to sell product n of the related manufacturer can be made. These costs, for example, include allocating a suitable stage in a retailer’s shop for representing the product. However, when M and R integrate, this item will be eliminated because they act as one. Mathematically, if we make the integrated function of M and R, marketing cost write-off from the equation. By considering the above rules, the final payoff function of the MR Player is described as (7):

$$Z_{MR} = TR_{MR} - TC_{MR}$$

$$TR_{MR} = \sum_{n=1}^N P_{r_n} \cdot D_n$$

$$D_n = k \cdot P^{-\alpha} r_n$$

$$TC_{MR} = TPC_{MR} + THC_{MR} + TBC_{MR} + TSC_{MR} + TCP_{MR}$$

$$THC_{MR} = \sum_{n=1}^N C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n}}, \quad TBC_{MR} = \sum_{n=1}^N \frac{C_B \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n}}, \quad TPC_{MR} = \sum_{n=1}^N u \cdot D_n^{1-\gamma}$$

$$TPC_{MR} = \sum_{n=1}^N [D_n \cdot \sum_{s=1}^M (k_{s_n} \cdot C_{p_s})], \quad TSC_{MR} = \sum_{n=1}^N [((\sum_{s=1}^M C_{o_{s_n}}) + C_{S_n}) \cdot \frac{D_n}{Q_{r_n}}]$$

$$\begin{aligned} \text{Max } Z_{MR} = & \sum_{n=1}^N [(P_{r_n} - (\sum_{s=1}^M k_{s_n} \cdot C_{p_s}) - u \cdot D_n^{-\gamma}) \cdot D_n] - \\ & \sum_{n=1}^N \left([C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n}}] + [\frac{C_B \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n}}] + [(\sum_{s=1}^M C_{o_{s_n}}) + C_{S_n}] \cdot \frac{D_n}{Q_{r_n}} \right) \end{aligned} \quad (7)$$

s.t :

$$D_n = k \cdot P_{r_n}^{-\alpha} > 0.$$

Decision variables for this function are $P_{r_n}, Q_{r_n}, B_n, C_{p_s}$ and constraints insist that demand should not be negative. The objective function maximizes the manufacturer and retailer when both act as one player. In this case, the overall income from selling the final product to costumers should be maximized while production, holding, stock out and purchasing costs should be minimized. As they act as one player, only a single warehouse is considered and shared by both for the final products.

3.4.2. Integration function (S-M Nash game) of the supplier – manufacturer

While suppliers and manufacturers are integrating, their coalition will affect their costs and benefits. In this situation, the only income will supply from the mass price from manufacturers to retailers. On the other hand, the costs of a new integrated level will include production costs, shortage costs, setup costs and holding costs. By considering the above rules, the final payoff function for the MR Player is described as (8):

$$Z_{SM} = TR_{SM} - TC_{SM}$$

$$TR_{SM} = \sum_{n=1}^N P_n \cdot D_n$$

$$D_n = k \cdot P_{r_n}^{-\alpha} \cdot C_{M_n}^{\beta}$$

$$TC_{SM} = TPC_{SM} + THC_{SM} + TBC_{SM} + TSC_{SM} + TCP_{SM}$$

$$THC_{SM} = \sum_{n=1}^N C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n}}$$

$$TBC_{SM} = \sum_{n=1}^N \frac{C_B \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n}}, \quad TPC_{SM} = \sum_{n=1}^N u \cdot D_n^{1-\gamma}$$

$$TPC_{SM} = \sum_{n=1}^N \sum_{s=1}^M (C_{S_o} \cdot k_{s_n}) \cdot D_n, \quad TSC_{SM} = \sum_{s=1}^M (C_{S_s} \cdot \sum_{n=1}^N \frac{D_n}{Q_{r_n}})$$

$$\begin{aligned}
 \text{Max } Z_{SM} = & \left[\left(\sum_{n=1}^N (P_n - u \cdot D_n^{-\gamma}) - \sum_{s=1}^M (C_{S_O} \cdot k_{s_n}) \cdot D_n \right) - \right. \\
 & \left. \left[\sum_{n=1}^N \frac{C_B \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n}} \right] - \left[\sum_{s=1}^M (C_{S_S} \cdot \sum_{n=1}^N \frac{D_n}{Q_{r_n}}) \right] - \left[\sum_{n=1}^N C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n}} \right] \right] \quad (8)
 \end{aligned}$$

s.t :

$$D_n = k \cdot P_n^{-\alpha} \cdot C_{M_n}^\beta > 0.$$

Decision variables for this function are $P_n, Q_{r_n}, B_n, P_{r_n}, C_{M_n}$ and constraints insist that demand should not be negative. The objective function maximizes the manufacturer and supplier when both act as one player. In this case, the overall income from the selling product to retailers should be maximized while production, holding, stock out and purchasing costs should be minimized.

3.5. Best responses

Based on the Nash definition of equilibrium, the best responses of each player should be estimated by others. As the objective function of each player is a nonlinear mathematical model, the best responses of each player, due to its decision variable, are calculated by partial differentiation and the first derivative. By considering the best response definition, continuous and discrete payoff functions, Nash equilibrium $N(G)$ is computable by the derivation of utility or the payoff function of each player $U_i(S_i, S_{-i}) = f(S_i, S_{-i})$ regarding a specific decision variable. The best response for the discrete function is calculated by (9) and that for continuous payoff functions is illustrated in (10) (Rasmusen 2005):

$$\begin{aligned}
 & \left. \begin{aligned} B_1(S_2^*) = S_1^* \\ B_2(S_1^*) = S_2^* \end{aligned} \right\} \rightarrow N(G) = \{S^* = (S_1^*, S_2^*)\} \\
 & \left. \begin{aligned} B_1(S_{-1}) = S_1^* \\ B_2(S_{-2}^*) = S_2^* \\ B_n(S_{-n}^*) = S_n^* \end{aligned} \right\} \rightarrow N(G) = \{S^* = (S_1^*, S_2^*, \dots, S_n^*)\} \quad (9)
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{aligned} U_1(S_1, S_2) = f(S_1, S_2) \\ U_2(S_1, S_2) = f(S_1, S_2) \end{aligned} \right\} \rightarrow \\
 & \left. \left\{ \begin{aligned} \frac{dU_1(S_1, S_2^*)}{dS_1} = 0 \rightarrow B_1(S_2^*) = f_1(S_2^*) \\ \frac{dU_2(S_1^*, S_2)}{dS_2} = 0 \rightarrow B_2(S_1^*) = f_2(S_1^*) \end{aligned} \right\} \right\} \rightarrow \left\{ \begin{aligned} B_1(S_2^*) = S_1^* \\ B_2(S_1^*) = S_2^* \end{aligned} \right. \quad (10)
 \end{aligned}$$

3.5.1. Best responses for the decentralized game

Each player in three-level supply chains will make the best decision when playing a game in the SC, i.e. they move simultaneously and there is no Stackelberg leader. Make a remark that our research is based on non-cooperative game situations; thus, in the game theory, orders and demand follow the pull system, but pricing and other agreements may not. For example, in Stackelberg games, leadership could happen at any level of the SC, which contradicts the pull system. The game theory is a pre-action approach and leads the players to order a quantity which maximizes their and other levels. While SC information and the demand function are completely accessible, any

Table 3. Best responses

Best response	Decision variables	Player
$P_{r_n}^* = \frac{\alpha.(P_n + C_{S_m} \cdot Q_{r_n}^{-1})}{\alpha - \beta - 1}$	(11)	C_{M_n}, P_{r_n} Retailers
$C_{M_n}^* = \frac{\beta.(P_n + C_{S_m} \cdot Q_{r_n}^{-1})}{\alpha - \beta - 1}$	(12)	
$P_n^* = \phi' . \left[\left(\sum_{s=1}^M (k_{s_n} \cdot C_{p_s}) + u \cdot D_n^{-\gamma} \right) + \left[\frac{\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n}}{Q_{r_n}} \right] + \left[C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \lambda_n \cdot Q_{r_n} \cdot D_n} \right] + \left[\frac{C_{B_n} \cdot B_n^2}{2 \lambda_n \cdot Q_{r_n} \cdot D_n} \right] \right]$	(13)	Q_{r_n}, P_n, B_n Manufacturers
$Q_{r_n}^* = \sqrt{\frac{2 \cdot \left(\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n} \right) \cdot D_n}{E_n \cdot \lambda_n \cdot C_{B_n}}}$	(14)	
$B_n^* = \sqrt{\frac{2 \cdot \left(\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n} \right) \cdot D_n \cdot E_n \cdot \lambda_n}{C_{B_n}}}$	(15)	
$C_{P_S}^* = \phi \cdot \left[C_{S_o} + \frac{\left(\sum_{n=1}^N \frac{D_n}{Q_{r_n}} \cdot C_{S_s} \right) + \left(\sum_{n=1}^N k_{S_s} \cdot C_{S_o} \cdot k_{S_n} \cdot \frac{Q_{r_n}}{2} \right)}{\sum_{n=1}^N k_{S_n} \cdot D_n} \right]$	(16)	C_{P_S} Suppliers

level enables to model the SC profit function and make the best decision. By considering the reasonable behaviour of each player and the Nash best response principle, the best decisions on each player in the three-echelon SC will be concluded by the derivation of the payoff function to decision variables. The first order condition of each payoff function is used for the best response, and the second condition is applied for concavity analysis. By calculating the determinant of the Hessian matrix of each player and with regard to its decision variables, it has been concluded that all models are concave to their decision variables, and therefore optimal solutions to the proposed models are definable. Table 3 represents the best response of each player if the Nash principle is used by calculating the first order condition of the payoff functions of each player regarding their decision variables. The details of the results presented in the table are mentioned in Appendixes 1 to 3.

3.5.2. Best MR responses

In this situation, two players or two levels of the supply chain are considered, i.e. suppliers and the MR level. Suppliers are similar to the decentralized model with the same best response. For a MR Player (it is assumed that MR becomes one unit), by a change in the payoff function, the best responses will vary as (17). We have to mention that integration affects the relation between manufacturers and retailers; hence, mass price from the manufacturer to the retailer as well as marketing cost will be eliminated from the final results:

$$\left[M \xleftrightarrow{\text{joint}} R \right] \xleftrightarrow{\text{versus}} S$$

$$\frac{\partial Z_{MR}}{\partial P_{rn}} = 0 \rightarrow P_{rn} = \frac{\alpha \cdot [(\sum_{s=1}^M C_{O_{sn}}) + C_{S_n}] + ((\sum_{s=1}^M k_{S_n} \cdot C_{P_S}) \cdot Q_n)}{Q_n \cdot (\alpha - 1)} ; \forall r, n \in N$$

$$\frac{\partial Z_{MR}}{\partial Q_n} = 0 \rightarrow Q_n = \sqrt{\frac{2 \cdot (\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n}) \cdot D_n}{\lambda_n \cdot E_n \cdot C_{B_n}}} ; \forall r, n \in N$$

$$\frac{\partial Z_{MR}}{\partial B_n} = 0 \rightarrow B_n = E_n \cdot \lambda_n \cdot Q_n ; \forall r, n \in N$$

$$C_{P_S} = \varphi_s \cdot [C_{S_o} + \frac{(\sum_{n=1}^N \frac{D_n}{Q_{r_n}} \cdot C_{S_s}) + (\sum_{n=1}^N k_{S_s} \cdot C_{S_o} \cdot k_{S_n} \cdot \frac{Q_{r_n}}{2})}{\sum_{n=1}^N k_{S_n} \cdot D_n}] ; \forall s \in M. \tag{17}$$

3.5.3. Best SM responses

In this case, two players or two levels of the supply chain are taken into account, i.e. retailers and the SM level. Retailers are similar to the decentralized model with the same best response. For a SM Player, by a change in the payoff function, the best responses will vary as (18). We have to mention that integration affects the relation between manufacturers and suppliers, and therefore mass price from the supplier to the manufacturer will be eliminated from the final results. In all situations, the demand function and production rate equality always affect the obtained results:

$$\begin{aligned}
 & \left[S \xleftrightarrow{\text{joint}} M \right] \xleftrightarrow{\text{versus}} R \\
 & \frac{\partial Z_{MS}}{\partial Q_{r_n}} = 0 \rightarrow Q_{r_n} = \sqrt{\frac{2 \cdot (\sum_{s=1}^M C_{S_s} + C_{S_n}) \cdot D_n}{\lambda_n \cdot E_n \cdot C_{B_n}}}; \quad \forall n \in N \\
 & \frac{\partial Z_{MS}}{\partial B_n} = 0 \rightarrow B_n = E_n \cdot \lambda_n \cdot Q_{r_n}; \quad \forall n \in N \\
 & P_n = \varphi_n' \cdot ([u \cdot D_n^{-\gamma} + \sum_{s=1}^M (C_{S_o} \cdot k_{s_n})]) + [\frac{C_B \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n} \cdot D_n}] + [\sum_{s=1}^M \frac{C_{S_s}}{Q_{r_n}}] + [C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n} \cdot D_n}] \\
 & P_{r_n} = \frac{\alpha \cdot (P_n + C_{S_{rn}} \cdot Q_{r_n}^{-1})}{\alpha - \beta - 1}; \quad \forall n \in N \\
 & C_{M_n} = \frac{\beta \cdot (P_n + C_{S_{rn}} \cdot Q_{r_n}^{-1})}{\alpha - \beta - 1}; \quad \forall n \in N \\
 & D_n = k \cdot P_{r_n}^{-\alpha} \cdot C_{M_n}^{\beta}; \quad \forall n \in N \\
 & \lambda_n = 1 - \frac{D_n}{PC_n}; \quad \forall n \in N.
 \end{aligned} \tag{18}$$

4. Numerical example

With reference to the methodology of our research and to the evaluated results, the verification and validation of the investigated outputs have been analysed applying to a numerical example of a hypothetical supply chain. Due to a lack of numerical and historical information, the design of the experimental approach has been performed to produce suitable data. Each experiment has been assessed by the proposed models, and the total profit of the SC has bred out based on coalition or decentralization circumstances. Following sensitivity analysis, the overall SC profits of each method have been compared, and finally, an appropriate approach has been suggested. Through a

numerical experiment, 17 design cases (through the performance of experiments) have been developed, and the total profit of the supply chain under each scenario has been evaluated. Statistical tests on the conducted 17 experiments have disclosed that the decentralized system performs significantly worse than the integration of the supplier with the manufacturer, whereas no significant difference in other combinations can be observed. Jia *et al.* (2013) used the same numerical example based on the Stackelberg game. They considered and compared three types of leadership and concluded that retailer leadership would beget the highest profit for the supply chain. Regarding the novelty of our research consisting of channel integration and the best responses of the proposed coalition, the above mentioned numerical example has been solved by our new non-cooperative game theory approach and coalition vs. decentralization. Conclusively, the profit achieved from the coalition was higher than employing decentralization and leadership methods.

4.1. Definition of the problem

Considering the above mentioned models for sensitivity analysis and leadership selection, the three-echelon supply chain, including 2 suppliers, 2 manufacturers and 2 retailers has been designed. Table 4 indicates the numerical amounts of parameters proposed in the supply chain.

Table 4. Initial data on the numerical example

Amount 2	Par M	Amount 2	Par R
4	$C_{S_r}(1)$	2	S
0.15	k_1'	5	$C_{S_r}(2)$
1.1	φ_1'	0.2	k_2'
3	$k_{sn}(11)$	1.15	φ_2'
3	$k_{sn}(21)$	4	$k_{sn}(12)$
6	$Co_{sn}(11)$	3	$k_{sn}(22)$
4	$Co_{sn}(21)$	5	$Co_{sn}(12)$
1	$C_B(1) = C_B(2)$	6	$Co_{sn}(22)$
25	$C_{S_s}(1)$	0.5	$C_{h_n}(1) = C_{h_n}(2)$
0.15	$k_{S_s}(1)$	24	$C_{S_s}(2)$
2	$C_{S_o}(1)$	0.2	$k_{S_s}(2)$
15	$PC(1) = PC(2)$	1.5	$C_{S_o}(2)$
8	$C_{S_n}(2)$	7	$C_{S_n}(1)$
1.1	φ_2	1.15	φ_1

For sensitivity analysis, five constants, including a , b , g , k , u , have been chosen. The lower and upper bounds of these five elements are shown in Table 5.

Table 5. Key parameters for sensitivity analysis

Max	Min	Par
1.25	1.2	α
0.15	0.05	β
0.1	0.01	γ
4000	3000	k
4	2	u

While using design of experiment (DOE) and 2^{k-p} experiments, and including one central point in each block, 17 different tests have been designed employing MINITAB 16.5 software (see Table 6).

Table 6. Design of experiment

β	α	k	u	γ	Design
0.15	1.2	4000	4	0.01	1
0.05	1.2	3000	2	0.1	2
0.05	1.2	3000	4	0.01	3
0.05	1.25	3000	4	0.1	4
0.15	1.25	4000	4	0.1	5
0.15	1.2	3000	4	0.1	6
0.1	1.225	3500	3	0.055	7
0.15	1.25	3000	4	0.01	8
0.15	1.25	3000	2	0.1	9
0.15	1.2	3000	2	0.01	10
0.05	1.25	3000	2	0.01	11
0.05	1.2	4000	4	0.1	12
0.05	1.2	4000	2	0.01	13
0.15	1.2	4000	2	0.1	14
0.15	1.25	4000	2	0.01	15
0.05	1.25	4000	4	0.01	16
0.05	1.25	4000	2	0.1	17

4.2. Results

All experiments designed in Table 5 are performed for the decentralization game, MR game and SM game. For the decentralized game, the models have been coded, debugged and solved by LINGO 11. On the other hand, for MR and SM games, multi nonlinear

equations have been solved using the Levenberg – Marquardt algorithm through fsolve application in MATLAB software. The calculated overall profit of the supply chain taking into account different types of problems is displayed in Table 7.

Table 7. The overall profit of the supply chain

Design	Decentralized	MR game	SM game
1	2288	1918	3406
2	1593	1939	1775
3	1552	1929	1737
4	1067	1849	1315
5	2442	1969	2407
6	2552	1866	2535
7	1995	1980	2073
8	1800	2462	1869
9	1819	1501	1839
10	2572	1493	2552
11	1088	1892	1320
12	2135	1451	2345
13	2168	1467	2380
14	2438	2503	3421
15	2498	1900	2474
16	1500	2441	1764
17	1507	2492	1794

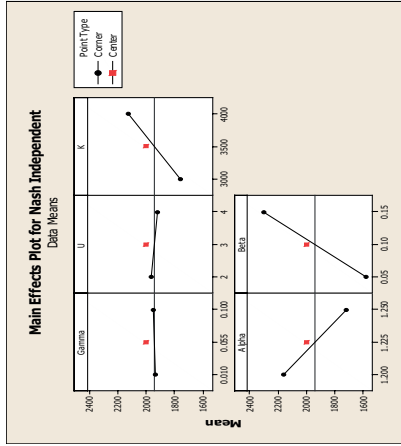
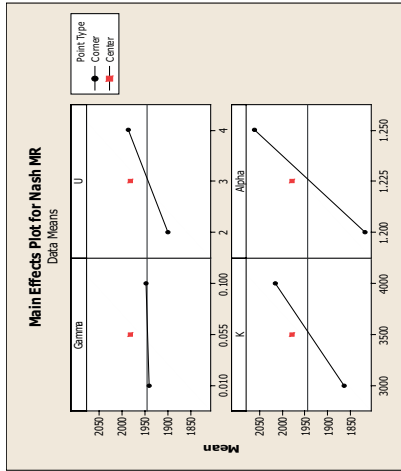
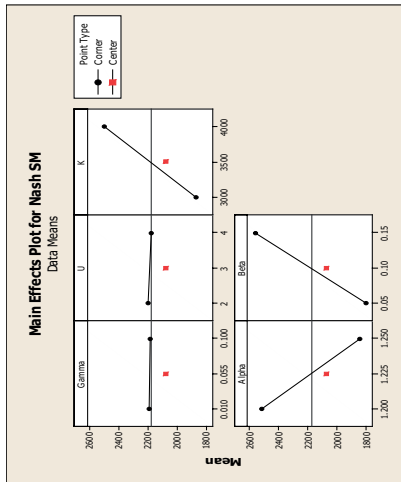
By taking a two-paired test, the results of the three types of decision making used in the supply chain have been compared. The findings obtained employing MINITAB 16.5 software is presented in Table 8. It is obvious that coalition, integration and semi centralization bring better results and profit to the SC.

Table 8. Comparison results from three coalition and independent games

P-Value	T-Value	St Dev	Mean	Experiments	Game type	Paired T test type
0.193	1.36	608	2177	17	Nash SM	Nash SM versus Nash MR
		355	1944	17	Nash MR	
		705	232	17	Difference	
0.988	-0.01	498	1942	17	Independent	Nash Independent versus Nash MR
		355	1944	17	Nash MR	
		672	-2	17	Difference	
0.009	-2.96	498	1942	17	Independent	Nash Independent versus Nash SM
		608	2177	17	Nash SM	
		327.4	-234.9	17	Difference	

Table 9. The analysis of three proposed models for SC behaviour

Decentralized Game		MR Game		SM Game	
P	Effect	Effect	Coefficient	Effect	Coefficient
Gamma	10.7	8.4	5.4	-8.6	-4.3
U	-43.1	87.2	-21.6	-21.9	-10.9
K	366.8	151.3	183.4	631.2	315.2
Alpha	-447.1	242.6	-223.1	-671.2	-375.3
Beta	724.9	*	362.4	759.3	379.4



The main effects of each situation considering five critical elements are calculated using MINITAB 16.5 software and shown in the figures of Table 9. In all models, Gama has the least noticeable effect, whereas K, Alpha and Beta have the strongest one, which means that changes in unit production cost will impact SC profit less than demand oscillation. K and Beta directly affect the overall profit of the SC in all three situations, and Alpha makes an impact inversely on decentralized and SM games. In the MR semi-integrated game, U parameter has direct effects on the overall profit of the SC. Thus, it can be concluded that higher marketing cost increases SC profit as a lower retailer price do for decentralized and SM games. Also, Beta parameter does not exist in the MR game, and therefore marketing cost is not included. As Beta parameter does not exist in the MR game, marketing cost is not included. We also conclude that changes in unit production cost will impact SC profit less than demand oscillation. In addition, higher marketing cost increases SC profit as a lower retailer price does for decentralized and SM games.

Conclusions

The conducted research has demonstrated coordination in multi-echelon supply chains in which the non-cooperative game theory approach is used as a suitable tool for coordinating pricing, inventory and marketing expenditure policies in the unlimited three-level supply chain, since a different level acts independently (decentralization) or, in some cases, integrates with other levels (coalition). For this matter, first of all, the objective function of each player and constraints has been modelled. Next, the best response of each player has been obtained based on the definition of the Nash equilibrium. Finally, two scenarios of decentralization and coalition have been modelled and analysed conducting an experiment. Among this, the concavities of the proposed models have been obtained and sensitivity analyses for different situation have been illustrated. A remark that a nonlinear demand and production cost function, besides unlimited levels and stock out situation taking into account the inventory system of the manufacturer, has been considered. Through the numerical experiment, 17 design cases (performed experiments) have been developed and the total profit of the supply chain under each scenario has been evaluated. According to statistical tests on the above introduced 17 experiments, the authors have found that the decentralized system performs significantly worse than the integration of the supplier with the manufacturer, whereas no significant difference can be observed for other combinations. To sum up, coalition and semi-integration circumstances are more effective and beget higher profit for the total supply chain. In addition, based on the considered SC, coalition between manufacturers with suppliers is more effective than that between retailers and manufacturers.

The above discussed situation and assumptions made in this paper are the keys for future researches. Taking into account more levels in the supply chain through distribution centres, warehouses or final stores will lead researchers to a comprehensive model for coordinating the supply chain in the future. For more accuracy and to provide the proposed models performable in reality and industrial situations, the case studies based on industrial information from the existing supply chains should be gathered and examined.

In conclusion, as the competency of information and complete information shared at different levels of the supply chain under real circumstances seem to be impossible, using incomplete or imperfect game theory approaches such as the signalling game or Nash Bayesian game will solve this problem and reach more realistic options in the future.

Appendixes

Appendix 1. Best retailer responses

$$\frac{\partial Z_r}{\partial P_{r_n}} = 0 \rightarrow [(1-\alpha) \cdot k \cdot P_{r_n}^{-\alpha} \cdot C_{M_n}^\beta] + [(\alpha \cdot k \cdot P_{r_n}^{-\alpha-1} \cdot C_{M_n}^\beta) \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})] = 0$$

$$\rightarrow \alpha \cdot P_{r_n}^{-\alpha-1} [(P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})] =$$

$$(1-\alpha) \cdot P_{r_n}^{-\alpha} \Rightarrow P_{r_n} (C_{M_n}) = \frac{\alpha \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})}{\alpha - 1},$$

$$Z_r(P_{r_n}, C_{M_n}) \xrightarrow{P_{r_n}=P_{r_n}(C_{M_n})} [k \cdot C_{M_n}^\beta \cdot \left(\frac{\alpha \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})}{\alpha - 1}\right)^{-\alpha} \cdot \left(\frac{\alpha \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})}{\alpha - 1} - (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})\right)] + \left[\frac{1}{2} \cdot Q_{r_n} \cdot k_n \cdot P_n\right]$$

$$\Rightarrow Z_r(C_{M_n}) = [k \cdot C_{M_n}^\beta \cdot \left(\frac{\alpha^{-\alpha}}{(\alpha - 1)^{1-\alpha}}\right) \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})^{1-\alpha}] + \left[\frac{1}{2} \cdot Q_{r_n} \cdot k_n \cdot P_n\right],$$

$$\frac{\partial Z_r(C_{M_n})}{\partial C_{M_n}} = 0 \rightarrow \left(k \cdot \frac{\alpha^{-\alpha}}{(\alpha - 1)^{1-\alpha}}\right) \cdot [(\beta \cdot C_{M_n}^{\beta-1} \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})^{1-\alpha}) +$$

$$((1-\alpha) \cdot C_{M_n}^\beta \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})^{-\alpha})] + 0 = 0$$

$$\rightarrow \beta \cdot C_{M_n}^{\beta-1} \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})^{1-\alpha} = (1-\alpha) \cdot C_{M_n}^\beta \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})^{-\alpha}$$

$$\rightarrow [\beta \cdot (P_n + C_{M_n} + C_{S_{r_n}} \cdot Q_{r_n}^{-1})] + [\beta \cdot C_{M_n}] =$$

$$(\alpha - 1) \cdot C_{M_n} \Rightarrow C_{M_n}^* = \frac{\beta \cdot (P_n + C_{S_{r_n}} \cdot Q_{r_n}^{-1})}{\alpha - \beta - 1},$$

$$P_{r_n}^* = P_{r_n}(C_{M_n}^*) \rightarrow P_{r_n}(C_{M_n}^*) = \frac{\alpha \cdot \left(P_n + \frac{\beta \cdot (P_n + C_{S_{r_n}} \cdot Q_{r_n}^{-1})}{\alpha - \beta - 1} + C_{S_{r_n}} \cdot Q_{r_n}^{-1}\right)}{\alpha - 1}$$

$$\Rightarrow P_{r_n}^* = \frac{\alpha \cdot (P_n + C_{S_{r_n}} \cdot Q_{r_n}^{-1})}{\alpha - \beta - 1}.$$

Appendix 2. Best manufacturer responses

$$\frac{\partial Z_n}{\partial B_n} = 0 \rightarrow 0 + 0 + 0 + [C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)}{\lambda_n \cdot Q_{r_n}}] - [\frac{C_{B_n} \cdot B_n}{\lambda_n \cdot Q_{r_n}}] = 0 \Rightarrow B_n = (\frac{C_{h_n}}{C_{h_n} + C_{B_n}}) \cdot \lambda_n \cdot Q_{r_n}$$

$$\frac{\partial Z_n}{\partial Q_{r_n}} = 0 \rightarrow 0 + \frac{(\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n}) \cdot D_n}{Q_{r_n}^2} + \frac{C_{B_n} \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n}^2} - \frac{C_{h_n} \cdot (2\lambda_n \cdot Q_{r_n}^2 - \lambda_n^2 \cdot Q_{r_n}^2 - B_n^2)}{2 \cdot \lambda_n \cdot Q_{r_n}^2} = 0$$

$$\Rightarrow [2 \cdot \lambda_n \cdot (\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n}) \cdot D_n] - C_{h_n} \cdot (2\lambda_n \cdot Q_{r_n}^2 - \lambda_n^2 \cdot Q_{r_n}^2 - B_n^2) + [C_{B_n} \cdot B_n^2] = 0,$$

$$[2 \cdot \lambda_n \cdot (\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n}) \cdot D_n] = Q_{r_n}^2 \cdot [-(C_{B_n} \cdot \lambda_n^2 \cdot (\frac{C_{h_n}}{C_{B_n} + C_{h_n}})^2) +$$

$$(2 \cdot C_{h_n} \cdot \lambda_n) - (C_{h_n} \cdot \lambda_n^2) - (C_{h_n} \cdot \lambda_n^2 \cdot (\frac{C_{h_n}}{C_{B_n} + C_{h_n}})^2)]$$

$$\xrightarrow{\text{if}} \frac{C_{h_n}}{C_{B_n} + C_{h_n}} = E_n \xrightarrow{\text{then}} Q_{r_n}^* = \sqrt{\frac{2 \cdot (\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n}) \cdot D_n}{E_n \cdot \lambda_n \cdot C_{B_n}}},$$

$$Z_n = [(P_n - \sum_{s=1}^M (k_{s_n} \cdot C_{p_s}) - u \cdot D_n^{-\gamma}) \cdot D_n] - [(\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n}) \cdot \frac{D_n}{Q_{r_n}}] - [C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n}}] - [\frac{C_{B_n} \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n}}] > 0$$

$$\rightarrow P_n^* > [\sum_{s=1}^M (k_{s_n} \cdot C_{p_s}) + u \cdot D_n^{-\gamma}] + [\frac{(\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n})}{Q_{r_n}}] + [C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n} \cdot D_n}] + [\frac{C_{B_n} \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n} \cdot D_n}],$$

$$P_n^* = \phi' \cdot [\sum_{s=1}^M (k_{s_n} \cdot C_{p_s}) + u \cdot D_n^{-\gamma}] + [\frac{(\sum_{s=1}^m (C_{O_{sn}}) + C_{S_n})}{Q_{r_n}}] + [C_{h_n} \cdot \frac{(\lambda_n \cdot Q_{r_n} - B_n)^2}{2 \cdot \lambda_n \cdot Q_{r_n} \cdot D_n}] + [\frac{C_{B_n} \cdot B_n^2}{2 \cdot \lambda_n \cdot Q_{r_n} \cdot D_n}] \text{ if } \phi' > 1.$$

Appendix 3. Best supplier responses

$$Z_S = [(C_{P_S} - C_{S_o}) \cdot \sum_{n=1}^N k_{s_n} \cdot D_n] - [\sum_{n=1}^N \frac{D_n}{Q_{r_n}} \cdot C_{S_s}] - [\sum_{n=1}^N k_{S_s} \cdot C_{S_o} \cdot k_{s_n} \cdot \frac{Q_{r_n}}{2}] > 0$$

$$\rightarrow C_{P_S}^* > C_{S_o} + \frac{[\sum_{n=1}^N \frac{D_n}{Q_{r_n}} \cdot C_{S_s}] + [\sum_{n=1}^N k_{S_s} \cdot C_{S_o} \cdot k_{s_n} \cdot \frac{Q_{r_n}}{2}]}{\sum_{n=1}^N k_{s_n} \cdot D_n}$$

$$\xrightarrow{\text{or}} C_{P_s}^* = \phi \cdot [C_{S_o} + \frac{(\sum_{n=1}^N \frac{D_n}{Q_{r_n}} \times C_{S_s}) + (\sum_{n=1}^N k_{S_s} \cdot C_{S_o} \cdot k_{S_n} \cdot \frac{Q_{r_n}}{2})}{\sum_{n=1}^N k_{S_n} \cdot D_n}] \quad \text{if } \phi > 1.$$

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