Routledge
Taylor & Francis Group
Edinburgh Napier UNIVERSITY
INTERNATIONAL JOURNAL OF STRATEGIC PROPERTY MANAGEMENT
ISSN 1648-715X print / ISSN 1648-9179 online
2011 Volume 15(4): 329–339
http://dx.doi.org/10.3846/1648715X.2011.631765

ON THE ANALYSIS OF PROPERTY UNIT SALES OVER TIME

David G. CARMICHAEL [1] ✉ and Maria C. A. BALATBAT [2]

[1] *School of Civil and Environmental Engineering, The University of New South Wales, Sydney 2052 NSW Australia*
E-mail: D.Carmichael@unsw.edu.au
[2] *Australian School of Business, The University of New South Wales, Sydney 2052 NSW Australia*
E-mail: M.Balatbat@unsw.edu.au

Received 25 August 2010; accepted 1 February 2011

ABSTRACT. Investors in multiple unit developments, such as apartments, townhouses, condominiums and connected dwellings, rely on a stream of sales in order to achieve a viable cash flow, a dominant issue. At the feasibility phase of a development, sales over time are estimated, and commonly this is done deterministically. However, with uncertainty in sales, such estimates may not be realised when the market is tested, and the investor's hoped-for cash flow may not be attained. Accordingly some acknowledgment of this uncertainty should be made in order to assess the associated risks. In this light, the paper presents an analysis of sales over time where uncertainty in sales is taken into account. The underlying model developed is based on Markov chains, specifically adapted to sales. Actual development data are used to illustrate the paper's approach and conclusions. The model provides useful core information on sales, both in quantum and timing, to the investor. The model provides a tool useful to practitioners, and one complementary to their existing sales analysis approaches. The paper provides an original contribution, and one of practical use, to established investor practices in the analysis of sales of multiple unit developments.

KEYWORDS: Analysis; Property sales; Uncertainty; Markov chains

1. INTRODUCTION

In the development of property involving multiple units, such as apartments, townhouses, condominiums and connected dwellings, the cash flow of an investor (property developer) is determined by unit sales. Such sales occur over time; there may be sales pre-construction, during construction and post-construction. An investor structures borrowings to match development expenditure minus any anticipated returns from sales.

Commonly an investor estimates or forecasts sales on a period-by-period basis. So for example, for each month pre-construction, each month during construction, and each month post-construction, sales estimates are made. Such estimates are based on experience, past developments and a 'feel' for the market. Workplace observations by the authors and discussions with industry personnel indicate that sensitivity, scenario analysis and simulation are used to address uncertainty by a few investors, but the majority of people do their estimates and any cash flow calculations deterministically.

Actual sales could be expected to not follow exactly sales estimates because of the many market uncertainties present. Complications arise when units fail to sell or only sell slowly, or where sales are delayed perhaps because of buyers being unable to arrange finance in time, or because buyers' financial situations change. The situation is repeated a number of times where there is staged development. Accordingly, some acknowledgment of the uncertainty or variability in sales estimates would be useful, and would appear essential input to any rational risk analysis.

The uncertainties that have to be taken into account in any analysis of an investor's sales and cash flow accordingly are the time and quantum variability of sales.

The paper shows that Markov chains readily handle the analysis of uncertain sales. States are defined as the period of time by which a unit remains unsold. Units that have been sold, and units that are not sold are given their own special states. Transitions between states reflect the selling characteristics of the development. The flow-type formulation of Markov chains is intuitively appealing, reflecting the way many investors think about sales over time; accordingly the approach should be understandable and readily acceptable to investors. The sales model, although based on underlying sophisticated mathematics, is found to be readily usable by practitioners without mathematical backgrounds, with no greater requirement than a spreadsheet to do the calculations.

Current investor practice on developments is to estimate and keep updated summaries of sales on a time basis, and this feeds into the business accounts. This paper provides an alternative, and it is believed more reliable source, of estimates for accounts purposes.

The paper's approach provides useful core information to the investor and for the investor's financial planning practices. Information on sales, both in quantum and timing, is provided in a ready fashion, while acknowledging uncertainty.

The paper provides an original contribution, and one of practical use, to established investor practices in the financial analysis of unit sales. The analysis output feeds into the investor's risk management practices and decision making; for example the investor may, as a result of the analysis findings, choose to change its promotion practices, construction schedule, market differently (including the use of different pricing strategies, such as discounting early 'off-plan' sales to create momentum, or pricing based on time-on-market), or perhaps consider alternative development configurations and timings.

The paper introduces the necessary Markov chain theory. This is then adapted to the sales case. Actual sales data are used to example the approach. Relevant background literature is reviewed.

The analysis presented in this paper is not restricted to any particular development, market conditions, construction or sales duration. It represents an approach that users can take and apply to their particular circumstances. The paper does not deal with uncertainties associated with construction expenditure and duration.

2. BACKGROUND

Risk related to property investment has attracted many researchers. Most publications in this area concentrate on completed developments and generated rental incomes, real estate investment trusts (REIT) and real estate investment portfolios. The reviews of Sirmans and Sirmans (1987), Hendershott and Haurin (1990), Norman et al. (1995) and Benjamin et al. (2001) show the extent of research in this area. The review of Benjamin et al. (2001) organises the literature into five categories: (i) returns on real estate investments; (ii) diversification and portfolio optimisation benefits of real estate; (iii) returns on real estate versus other types of investment; (iv) returns on real estate measured by REIT performance; and (v) inflation and real estate returns.

This paper distinguishes itself from this body of literature by looking at issues prior to development completion. With a similar distinction, Wurtzebach and Kim (1979) address costs prior to development completion using a Markov process, while Peiser (1984a), on land development, and MacFarlane (1995) opt for simulation. Peiser (1984 b) and Peiser and Chang (1999) discuss new town development. Whipple (1988, p. 91) notes "The evaluation of real estate development projects has produced a sparse literature. This is in sharp contrast to the voluminous output devoted to the analysis of income earning properties."

No authors appear to address the quantitative analysis of sales, including any attendant uncertainty, as is advanced in this paper. The range of risk issues in property development is discussed, among others, by Bannerman (1993), Rodney and Venmore-Rowland (1996), Byrne (1996, 1997), Newell and Steglick (2006) and Reymen et al. (2008). Pre-selling issues are discussed, among others, by Ong (1999) and Leung et al. (2007).

Markov chain theory is well established. The body of available literature is very large. There are numerous texts on the subject and even more numerous papers applying the theory to diverse applications; for example, Howard (1971), Taylor and Karlin (1994), Norris (1998), Yin and Zhang (2005), Howard (1960), Carmichael (1987) and Isaacson and Madsen (1976). This paper also draws inspiration from the state definition of Cyert et al. (1962) and Corcoran (1978). The Markov chain literature has developed a significant body of fruitful theory. Only part of Markov chain theory is implemented and examined in this paper.

3. MODELLING

Established standard Markov chain theory is first given by way of background. The peculiarities that have to be introduced in order to deal with sales are then advanced.

Existing Markov chain theory

Markov chains are described in terms of states, state transitions and probabilities attached to these. The transitions occur at discrete points in time. Howard (1960) likens Markov chains to frogs jumping between lily pads; the lily pads are the states, while the jumps are the transitions.

Let the probability of being in any state k, k = 1, 2, ..., m, be denoted π_k, Let p_{jk} be the probability of transition between states j and k in a given time period; j, k = 1, 2, ..., m. Define a row vector π with components π_k, and a matrix P with components p_{jk}. It follows from established Markov theory that,

$$\pi = \pi P \tag{1}$$

with

$$\sum_{k=1}^{m} \pi_k = 1 \tag{2}$$

Equations (1) and (2) present m + 1 equations in m unknowns, π_k, k = 1, 2, ..., m.

The transition probabilities have the properties,

$$\sum_{k=1}^{m} p_k = 1 \qquad j = 1, 2, ..., m \tag{3}$$

and

$$0 \le p_{jk} \le 1 \qquad j, k = 1, 2, ..., m \tag{4}$$

The sales model advanced below has both transient states (states that can be both entered and exited) and absorbing states (states that can be entered but not exited). Where r absorbing states are present, standard Markov chain treatments rearrange the rows and columns of P, such that the first r rows and first r columns of P correspond to the absorbing states, the remaining rows and columns refer to transient states, and P is partitioned into submatrices,

$$P = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix} \tag{5}$$

The r × r identity matrix I corresponds to transitions between absorbing states. 0 is an r × (m – r) zero matrix, because transitions from absorbing states to transient states are not possible. Transitions between transient states and absorbing states are captured in the (m – r) × r matrix R, while transitions between transient states are captured in the (m – r) × (m – r) matrix Q.

A fundamental matrix N, of size (m – r) × (m – r), can be defined,

$$N = (1 - Q)^{-1} \tag{6}$$

N gives the number of time periods that the process spends in each state before being absorbed. The (m – r) × r product matrix NR gives the probability of absorption in each of the absorbing states. The sum of each row of NR equals 1, that is the process must end up in the absorbing states.

Adaptation to sales

Consider now the adaptation of existing Markov chain theory described above to deal with sales. States are defined as the number of units unsold beyond period i, i = 0, 1, 2, ... That is, the states reflect the sales over time. The time period may typically be months or weeks (regular intervals), but any time unit can be chosen to suit the intended purpose of the calculations.

The state describing units unsold beyond i = n is referred to here as 'Unsold'. The value of n can be selected as appropriate to the development. It may represent, for example, the point in time at which the sales process gets transferred to something separate to that used for the majority of sales. Each investment analysis may choose to use a different n value.

An additional state is introduced covering units sold. Referred to here as 'Sold', it is denoted n'.

Both 'Unsold' and 'Sold' states are absorbing states (states that can be entered but not exited). That is, m = n + 2 and r = 2. The remaining states 0, 1, …, n-1 are transient states (states that can be both entered and exited).

The states and transitions between the states can be drawn as in Figure 1. The states are represented by circles, and the transitions by the arrows between the states. Adjacent to the arrows are shown transition probabilities, p_{jk}, j, k = 0, 1, 2, ..., n, n'.

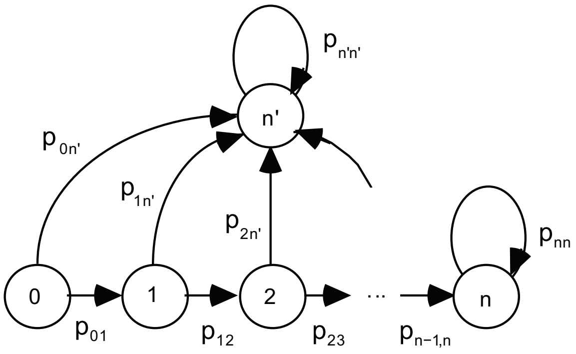

Figure 1. Sales transition diagram

Equation (1) may be obtained by equating the inputs and outputs of the states in Figure 1.

Sales occur in varying numbers and at varying times. Transition probabilities may be calculated from historical sales data, or estimated based on known development and market conditions, past experience or industry knowledge of likely sales for the type of development proposed, including the influence of price on 'time-on-market'. They represent the probabilities that units unsold by the end of one period will be unsold by the next period. Such estimates will depend on the time period chosen for the analysis. Let α_{jk} be the number of units unsold in state k that are transferred from state j (j, k = 0, 1, 2, ..., n, n') between periods i and i+1. Let p_{jk} be the probability associated with this transition. Then p_{jk} can be calculated as,

$$p_{jk} = \frac{\alpha_{jk}}{\sum_{k=0}^{n'} \alpha_{jk}} \quad j, k = 0, 1, 2, ..., n, n' \qquad (7)$$

But other methods, such as maximum likelihood, could be used to estimate p_{jk}. Because of the selected state definition, many of the transition probabilities will have values of zero, and the matrix P will only be sparsely populated. For example, for the absorbing states, $p_{n'n'} = 1$, $p_{n'0} = 0$, $p_{n'1} = 0$, ..., $p_{n'n} = 0$ and $p_{nn'} = 0$, $p_{n0} = 0$, $p_{n'1} = 0$, ..., $p_{n'n} = 1$.

Staged development

Consider now staged developments, where units are constructed in batches.

At any given time, for any given combination of numbers of units in each of the age categories, the matrix NR allows the investor to calculate the number of units that could be anticipated to end up being sold, or to end up unsold. Let the values (number of units unsold) in each age category k, k = 0, 1, 2, ..., n – 1, at any time i be denoted by a row vector V with elements $[V_0, V_1, ..., V_{n-1}]$, then

$$E['Sold'] = \sum_{s=0}^{n-1} V_s (NR)_s^1 \qquad (8)$$

$$E['Unsold'] = \sum_{s=0}^{n-1} V_s (NR)_s^2$$

where: E[ ] denotes expected value, and $(NR)^c$ denotes the first (c = 1) or second (c = 2) column values of NR.

The steady state numbers of units in each of the age categories are of interest. Let w units be made available for sale each time period. That is, w units enter state 0 each time period. The other states only take new values when transitions between states occur. That is, new additions (other than transitions) to states 1, 2, ..., n-1 are zero. The steady state values of units for sale in each state 0, 1, 2, ..., n – 1 is $w(N)_1$ where $(N)_1$ denotes the first row of N. The steady state values in states n' and n, that is the 'Sold' and 'Unsold' states, is $w(NR)_1$, where $(NR)_1$ denotes the first row of NR.

Mixed development

Where a development comprises a collection of unit types of quite different natures, appealing to differing markets and offered at differing prices, the analysis will give better results where this heterogeneity is acknowledged. That is, the different unit types are treated separately, rather than combining by assuming homogeneity. This is so because the sales frequencies will be quite different, and the impact on cash flow will be different.

4. SUMMARY APPROACH FOR AN INVESTOR

The model is intended to be applied at the feasibility stage of a development. The analysis feeds into the investor's cash flow calculations, and overall investment risk analysis.

The model can provide information that is useable by the investor in managing its financial affairs. It might, for example, point the way to whether an investor should change its promotion practices, construction schedule, market differently (including the use of different pricing strategies such as discounting early 'off-plan' sales to create momentum, or pricing based on time-on-market), or perhaps consider alternative development configurations and timings. The analysis feeds into the investor's accounting procedures.

The above development is summarized for the purpose of an investor implementing the model in practice. No more than a spreadsheet is needed to perform the matrix calculations.

1. Decide on a relevant time period. If an investor's borrowings are geared to monthly or weekly periods, then these would be appropriate time periods. But any time period can be chosen.
2. Decide on how many time periods, n, must pass before the investor concedes that a

sale may not be forthcoming, or the time at which the investor transfers the sales process to something separate to that used for the majority of sales. n is chosen to reflect the particular development.

3. Based on past developments or investor opinion, estimate the probabilities (or use historical data) that sales not made by the end of one period will still not have been made by the next period. This gives the p_{jk} terms that go to make up the P matrix, which in turn gives the R and Q matrices. The estimates will depend on the time period chosen.

   Some numerical studies undertaken by the authors on a range of development data indicate that the model results are relatively insensitive to small changes in the underlying assumptions including probability estimates. Accordingly, indicative estimates of past sales or reasonable opinion appear to be sufficient without needing precise estimates. For example, for the case considered below, a ±10% change in the transition probabilities p_{jk} gives changes in NR and N of the same order of magnitude, implying no real sensitivity.
4. Calculate N (Equation 6) and NR. The row sums of N give the average number of periods, after starting in the state corresponding to the row, before being classed as 'Sold' or 'Unsold'. The first column of NR gives the probabilities of units being sold. The second column of NR gives the probabilities of units not being sold and needing resolution.
5. Calculate E['Sold'] and E['Unsold'] (Equations 8). These are the number of units that could be anticipated to end up being sold, or to end up needing to be resolved, for any given combination of values in each of the age categories at any given time. This can be used as a reasonableness check or audit on conventional accounting practices where estimates are made on expected sales/non-sales in each age category.

Example calculations are given below applied to a case study. Calculations can be done using the matrix functions on a spreadsheet. No knowledge of the underlying Markov chain theory is necessary.

5. CASE EXAMPLE

Sales data from a development involving residential units on a greenfield site, are given in Table 1. In total, 95 units were constructed over 8 stages. All units were very similar, that is the group of units was effectively homogeneous, and all units may be analysed together. Table 1 sums the sales for the 8 stages in order to give a larger sample for calculation purposes, but the stages could have been treated separately. The construction duration for each stage, ranging from 11 to 16 units, was common at 5 months.

The data are given relative to the start of construction, but other reference dates could be used. The time measurement period is given as months, but other measurement periods (for example, weeks) could be used.

It is noted that some units were sold preconstruction, the majority sold during construction, while some units remained unsold on completion of the construction. Those remaining unsold were transferred to a sales process separate to that used for the majority of the sales (involving different selling agents and different marketing).

Table 1. Case example. Sales relative to construction commencement date

Period	Sales	Unsold after given period
		95
Pre construction start (presales)	14	81
1st month of construction	14	67
2nd month of construction	21	46
3rd month of construction	17	29
4th month of construction	14	15
5th (last) month of construction	10	5

Sales are tracked according to monthly periods. Period i = 0 is defined as the time selling commences, and periods i = 1, 2, 3, ... represent following months. Here, i = 1 corresponds to the start of construction, that is, sales started pre-construction.

Units unsold beyond n = 6 months are taken as representing something for transfer to a sales process separate to that used for the majority of the sales.

From the data, $p_{j,j+1}$, j = 0, 1, 2, ..., n − 1, are 0.853, 0.827, 0.687, 0.630, 0.517, and 0.333. The remaining transition probabilities may be obtained by equating the inputs and outputs of the states in Figure 1.

Populating the P matrix with these transition probabilities, and partitioning P into R and Q matrices gives:

$$R = \begin{bmatrix} 0.147 & 0 \\ 0.173 & 0 \\ 0.313 & 0 \\ 0.370 & 0 \\ 0.483 & 0 \\ 0.667 & 0.333 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 0 & 0.853 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.827 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.687 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.630 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.517 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix N gives information on the number of months that unsold units spend in each state and the number of months before unsold units end up being sold.

From Equation (6),

$$N = (1 - Q)^{-1} =$$

$$\begin{bmatrix} 1 & 0.853 & 0.705 & 0.485 & 0.305 & 0.158 \\ 0 & 1 & 0.827 & 0.568 & 0.358 & 0.185 \\ 0 & 0 & 1 & 0.687 & 0.433 & 0.224 \\ 0 & 0 & 0 & 1 & 0.630 & 0.326 \\ 0 & 0 & 0 & 0 & 1 & 0.517 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The first row of N is 1, 0.853, 0.705, 0.485, 0.305 and 0.158 (sum of 3.51). That is, units unsold starting in state 0 (total unsold at the start of the selling process), will remain in this state an average of 1 month, will be in the state 'unsold after construction starts' an average of 0.853 months, will be in the state 'unsold 1 month after construction starts' an average of 0.705 months, and will be in the following states (2, 3, ... months after construction starts) an average of 0.485, 0.305 and 0.158 months.

Other information useful for planning purposes is that related to the time before units are sold or classed as unsold. The row sums of N give this information. In particular, the row sums give the average number of months, after starting in the state corresponding to the row, before being absorbed, that is before being classed as 'Sold' or 'Unsold'. For illustration, N has a first row sum of 3.51. That is, starting in state 0 (the start of the selling process), the average number of months before a unit is classed as 'Sold' or 'Unsold' is 3.51.

The matrix NR follows.

$$NR = \begin{bmatrix} 0.947 & 0.053 \\ 0.938 & 0.062 \\ 0.925 & 0.075 \\ 0.892 & 0.108 \\ 0.828 & 0.172 \\ 0.667 & 0.333 \end{bmatrix}$$

The first column of NR gives the probabilities of units in each of the age categories ending up in the state 'Sold', n', that is the probabilities of being sold. The second column of NR gives the probabilities of units in each of the age categories ending up in the state 'Unsold', n, that is the probabilities of ending up being unsold. The sum of each row of NR equals 1, that is a unit must either end up being sold or unsold by the nominated date. See Figure 2.

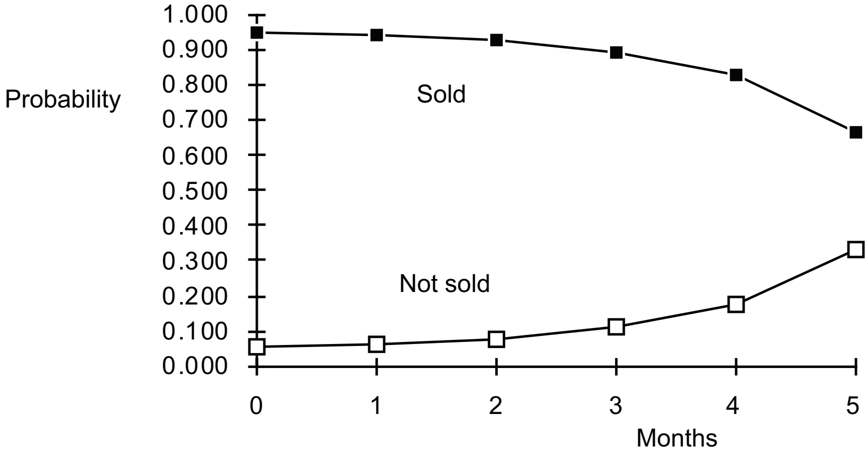

Figure 2. Case example. Probability of units up being sold/not sold

Staged development

Consider now the example development in its staged form, where the units are constructed in batches averaging 12, and a new construction stage is started monthly. The same P matrix as above is assumed to carry over to staged construction.

For illustration purposes, assume that at some stage during the total development, the number of units in each of the age categories k = 0, 1, 2, ..., 5 is V = [6, 9, 12, 10, 7, 5] or a total of 49. Then the expected number of units that could end up being sold, E['Sold'], or unsold, E['Unsold'], by the nominated date are, from Equations (8), 43.3 and 5.7 respectively.

With w = 12 new units available monthly, the steady state values for each time period in each of the age categories 0, 1, 2, ..., n – 1 are [12.0, 10.2, 8.5, 5.8, 3.7, 1.9]. The steady state values in each time period for 'Sold' and 'Unsold' by the nominated date are [11.4, 0.6]; each month this is the number of units entering 'Sold' and 'Unsold' states.

6. CONCLUSION

The results derived from a Markov chain model of sales over time provide a very useful and practical tool for investors. Based on no more than estimates of sales, and using no more than the matrix functions available in spreadsheets, the results feed directly into investment management practices. Typically an investor might carry out the calculations at the time of undertaking a feasibility study.

Collectively, the data provided by the analysis give a useful platform for investment decision making, on matters such as staged development, number and spacing of stages, number of units and so on. The approach allows investors to manage their affairs, through providing specific information on:

- Expected sales over time.
- The impact of sales over time.
- Expected number of units sold and units classed as unsold.
- For staged development, the steady state age distribution of unsold units.

The information will assist in pointing the way to how an investor will bring units onto the market, in terms of number and timing.

The number of states used in the calculations will vary with the size of the time period chosen and with differing information expectations from the investor. The number of states, however, does not change the nature of the calculations. Having more states only means larger matrices in the spreadsheet computations, with little additional analysis, and no increase in difficulty.

Future research

The results presented in this paper are based on the assumption of constant transition probabilities. This assumption could be relaxed, but is believed to be reasonable based on the underlying data. It is also commented that many of the tractable features of the results in this paper would not be available should alternative assumptions be adopted; it then becomes a trade off between perhaps a slight increase in accuracy of assumptions versus computational complexity. Within each development, the assumption of constant transition probabilities would appear reasonable. It is only anticipated that sales characteristics will change over the duration of an individual development, for example, where there is a significant change in market conditions. With the present analysis, perhaps the best way to incorporate significant changing market conditions, is to do upper and lower bound analyses. However, between developments of different type and in different localities, it is expected that different transition matrices will apply. Nevertheless, future research could empirically examine this assumption on differing data sets for differing developments.

The paper makes reference to the matrix calculations being performed on a spreadsheet. This is suitable for one-off calculations. However there would be value in developing some generic software for use by investors. Such software would have standard inputs of time period, transition probabilities and so on, and would produce ready-to-use output of sales over time etc, for input to feasibility studies. The software might also include a database of typical development profiles that could be applied in feasibility studies without the need for the user to make specific development estimates.

The Markovian property implies that future states are determined by the present state only, and not earlier states. It permits a tractable analysis over more general stochastic processes, while offering a good representation of actual behaviour such as that described in the case example data presented above. But future research could test this property.

An indication of the robustness of the approach can be gained from looking at the sensitivity to changes in some of the underlying assumptions. Preliminary analysis on the case example data suggests that the analysis is not overly sensitive to underlying assumptions. There would be value in looking at other data sets applying to other types of developments, in order to confirm this conclusion on sensitivity. Specific issues could also be tested, for example, how do the results change with changing volume of units coming onto the market at any one time.

REFERENCES

Bannerman, S. (1993) A framework for improved development appraisals in developing countries, *Journal of Property Research*, 10(2), pp. 135–145. doi:10.1080/09599919308724086

Benjamin, J. D., Sirmans, G. S. and Zietz, E. N. (2001) Returns and risk on real estate and other instruments: more evidence, *Journal of Real Estate Portfolio Management*, 7(3), pp. 183–214.

Byrne, P. (1996) *Risk, uncertainty and decision-making in property development*. E&FN Spon, London.

Byrne, P. (1997) Fuzzy DCF: a contradiction in terms, or a way to better investment appraisal?, *RICS Research*, 19 pp., The Royal Institution of Chartered Surveyors, London.

Carmichael, D. G. (1987) *Engineering queues in construction and mining*. Ellis Horwood Ltd. (John Wiley and Sons), Chichester.

Corcoran, A. W. (1978) The use of exponentially-smoothed transition matrices to improve forecasting of cash flows from accounts receivable, *Management Science*, 24(7), pp. 732–739. doi:10.1287/mnsc.24.7.732

Cyert, R. M., Davidson, H. J. and Thompson, G. L. (1962) Estimation of the allowance for doubtful accounts by Markov chains, *Management Science*, 8(3), pp. 287–303. doi:10.1287/mnsc.8.3.287

Hendershott, P. H. and Haurin, D. R. (1990) Research on real estate investment, *Real Estate Economics*, 18(4), pp. 369–375. doi:10.1111/1540-6229.00528

Howard, R. A. (1960) *Dynamic programming and Markov processes*. Technology Press, Massachusetts Institute of Technology, Cambridge.

Howard, R. A. (1971) *Dynamic probabilistic systems, Volume I: Markov models*. Wiley, New York.

Isaacson, D. L. and Madsen, R. W. (1976) *Markov chains, theory and applications*. Wiley, New York.

Leung, B. Y. P., Hui, E. C. M. and Seabrooke, B. (2007) Asymmetric information in the Hong Kong forward property market, *International Journal of Strategic Property Management*, 11(2), pp. 91–106. doi:10.1080/1648715X.2007.9637563

MacFarlane, J. (1995) The use of simulation in property investment analysis, *Journal of Property Valuation and Investment*, 13(4), pp. 25–38. doi:10.1108/14635789510099427

Newell, G. and Steglick, M. (2006) Assessing the importance of property development risk factors, *Pacific Rim Property Research Journal*, 12(1), pp. 22–37.

Norman, E. J., Sirmans, G. S. and Benjamin, J. D. (1995) The historical environment of real estate returns, *Journal of Real Estate Portfolio Management*, 1(1), pp. 1–24.

Norris, J. R. (1998) *Markov chains*. Cambridge University Press, New York.

Ong, S. -E. (1999) Caveat emptor: adverse selection in buying properties under construction, *Property Management*, 17(1), pp. 49–64. doi:10.1108/02637479910255749

Peiser, R. B. (1984a) Risk analysis in land development, *Real Estate Economics*, 12(1), pp. 12–29. doi:10.1111/1540-6229.00308

Peiser, R. (1984b) Financial feasibility models in new town development, *The Town Planning Review*, 55(1), pp. 75–90.

Peiser, R. B. and Chang, A. C. (1999) Is it possible to build financially successful new towns? The Milton Keynes experience, *Urban Studies*, 36(10), pp. 1679–1703. doi:10.1080/0042098992773

Reymen, I. M. M. J., Dewulf, G. P. M. R. and Blokpoel, S. B. (2008) Framework for managing uncertainty in property projects, *Building Research and Information*, 36(6), pp. 580–592. doi:10.1080/09613210802214259

Rodney, W. and Venmore-Rowland, P. (1996) Private versus public sector property development projects: risk evaluation techniques, 26 pp., *RICS Research*, The Royal Institute of Chartered Surveyors, London.

Sirmans, G. S. and Sirmans, C. F. (1987) The historical perspective of real estate returns, *Journal of Portfolio Management*, 13(3), pp. 22–31. doi:10.3905/jpm.1987.409109

Taylor, H. M. and Karlin, S. (1994) *An introduction to stochastic modelling*. Academic Press, San Diego.

Whipple, R. T. M. (1988) Evaluating development projects, *Journal of Property Valuation and Investment*, 6(3), pp. 253–286. doi:10.1108/eb008029

Wurtzebach, C. H. and Kim, K. S. (1979) An investment analysis and decision making framework for real estate development, *Real Estate Economics*, 7(3), pp. 410–426. doi:10.1111/1540-6229.t01-17-00201

Yin, G. and Zhang, Q. (2005) *Discrete-time Markov chains*. Springer, New York.

SANTRAUKA

NEKILNOJAMOJO TURTO VIENETO PARDAVIMO KAINOS ANALIZĖ LAIKO ATŽVILGIU

David G. CARMICHAEL, Maria C. A. BALATBAT

Investuojantys į tokio nekilnojamojo turto statybas, kaip daugiabučiai, blokuoti namai ir namų kompleksai, tikisi sklandaus pardavimo, nes turi susidaryti pakankami pinigų srautai, ir tai yra svarbiausias klausimas. Rengiant būsimų statybų galimybių studiją, vertinamas pardavimas bėgant laikui, ir tai paprastai daroma deterministiškai. Tačiau pardavimo apimtis nėra užtikrinta, tad, bandant rinką, nustatytos vertės gali likti nepasiektos, investuotojai gali negauti tokių pinigų srautų, kokių tikėjosi. Taigi, norint įvertinti susijusią riziką, reikia atsižvelgti į tokį netikrumą. Dėl to straipsnyje pateikiama pardavimo bėgant laikui analizė, kurioje atsižvelgiama į tai, kad pardavimo apimtis nėra užtikrinta. Pagrindinis sukurtas modelis grindžiamas Markovo grandinėmis, kurios specialiai pritaikomos pardavimui. Darbe pateikiamas požiūris ir išvados pristatomos naudojant realius statybų duomenis. Investuotojui modelis suteikia esminę naudingą informaciją apie pardavimą – tiek apie kiekį, tiek apie laiką. Modelis bus parankus praktikams ir papildys turimus pardavimo analizės būdus. Originalus ir praktinis darbo indėlis papildys įprastus metodus, kuriuos investuotojai taiko analizuodami būsto pardavimą.