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EVALUATION OF THE RECENT HIGH-DEGREE COMBINED GLOBAL GRAVITY-FIELD MODELS FOR GEOID MODELLING OVER KENYA

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Abstract. This study carries out an evaluation of the recent high-degree combined global gravity-field models (EGM2008, EIGEN-6C4, GECO and SGG-UGM-1) over Kenya. The evaluation is conducted using observed geoid undulations (18 data points, mainly in Nairobi area) and free-air gravity anomalies (8,690 data points, covering the whole country). All the four models are applied at full spherical harmonic degree expansion. The standard deviations of the differences between observed and GGMs implied geoid undulations at 18 GPS/levelling points over Nairobi area are ±11.62, ±11.48, ±12.51 and ±11.75 cm for EGM2008, EIGEN-6C4, GECO and SGG-UGM-1, respectively. On the other hand, standard deviations of the differences between observed and GGMs implied free-air gravity anomalies at 8,690 data points over Kenya are ±10.11, ±10.03, ±10.19 and ±10.00 mGal for EGM2008, EIGEN-6C4, GECO and SGG-UGM-1, respectively. These results indicate that the recent high-degree global gravity-field models generally perform at the same level over Kenya. However, EIGEN-6C4 performs slightly better than EGM2008, GECO and SGG-UGM-1, considering the independent check provided by GPS/levelling data (admittedly over a small area). These results further indicate a good prospect for the development of a precise gravimetric geoid model over Kenya using EIGEN-6C4 by integrating local terrestrial gravity data in a remove-compute-restore scheme.

Keywords: geoid undulation, free-air gravity anomaly, GPS, precise levelling, global gravity-field model.

Introduction

High-degree combined global gravity-field models (GGMs) are realized from optimal combination of satellite, terrestrial and altimetry-derived gravity datasets. The optimization through least squares techniques produce spherical representation of global gravity-field models by spherical harmonic coefficients. The choice of a suitable GGM for local/regional/global geoid modelling must be based on objective validation results. The high-degree combined global gravity-field models evaluated in this study include EGM2008 (Pavlis et al., 2012), EIGEN-6C4 (Förste et al., 2014), GECO (Gilardoni et al., 2016) and SGG-UGM-1(Liang et al., 2018). These models are principally developed from GOCE data (in the long-to-medium wavelength components) and EGM2008 (in the medium-to-short wavelength components). Although LAGEOS data are also included in the development of EIGEN-6C4.

Th improvements provided by GOCE data in the long-to-medium wavelength components have been assessed and discussed by various authors (Gruber et al., 2011; Hirt et al., 2011; Cheng & Ries, 2015; Godah et al., 2015; Huang & Véronneau, 2015; Alothman et al., 2016; Odera & Fukuda, 2013, 2017; Odera, 2019). The recent high-degree combined GGMs are developed to high spherical harmonic degrees (EGM2008, EIGEN-6C4 and GECO ~up to 2,190 and SGG-UGM-1~up to 2,159). All the four models have not been evaluated in Kenya, except EGM2008 but only in Nairobi area (Odera, 2016). Recent evaluations of these models in other parts of the world can be found in Kostelecký et al. (2015) (EGM2008 and EIGEN-6C4), Gilardoni et al. (2016) (EGM2008, EIGEN-6C4 and GECO), and Liang et al. (2018) (EGM2008 and SGG-UGM-1).

Geoid modelling over Kenya has been done mainly by low and medium resolution global gravity-field models, GEM10B~upto degree–36 (Gachari & Olliver, 1986) and OSU91A~up to degree–360 (Gachari & Olliver, 1998). The most recent geoid model over Kenya, covering Eastern African region (Gachari & Olliver, 1998) was developed using OSU91A (Rapp et al., 1991) as a reference geopotential model, combined with terrestrial gravity and satellite altimetry data by applying remove-compute-restore procedure (Gachari & Olliver, 1998). An accuracy of this geoid

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*Corresponding author. E-mail: *patroba.odera@uct.ac.za* model was obtained as ±88 cm, from assessment using 25 Doppler observations (mostly over Kenya). This accuracy was achieved after removing the biases in the differences between Doppler/levelling and gravimetric geoid undulations. The accuracy of Doppler data was ±1 ~ ±3 m while levelling data was obtained by trigonometric heighting (Gachari & Olliver, 1998), indicating a lack of accurate data for validating a gravimetric geoid model over Kenya in 1998.

The current high-degree GGMs would provide a better reference gravity-field for developing a more accurate geoid model over Kenya. However, the choice of an appropriate GGM from the existing ones can only be made through an evaluation. This paper carries out an assessment of the current high-degree GGMs (EGM2008, EIGEN-6C4, GECO and SGG-UGM-1) using observed free-air gravity anomalies (over Kenya) and GPS/levelling geoid undulations (over Nairobi area). As it is known, GGMs only estimate height anomalies, hence, a conversion to geoid undulations is necessary. This is achieved by a conversion model complete to degree and order 2,160 (Pavlis et al., 2012). The levelling data used in the current study were obtained through a precise levelling procedure, hence, more accurate than the trigonometric heights used in the previous studies.

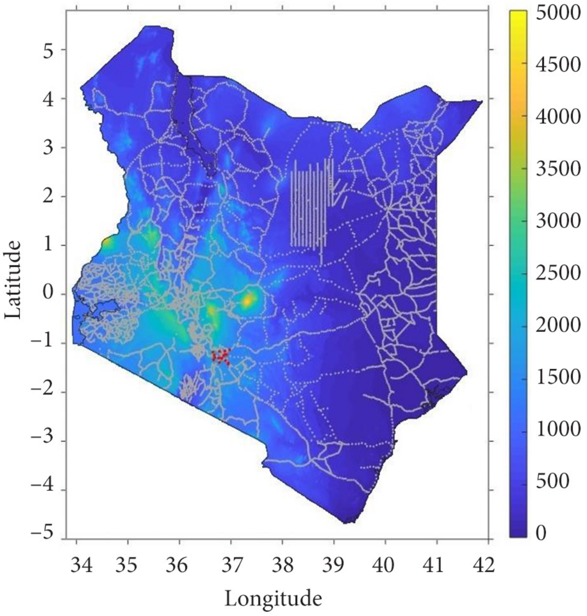

Figure 1. Spatial distribution of GPS/levelling (red dots) and first order gravity (gray dots) data points over Kenya, with the general topography of Kenya from SRTM data in the background (elevation units are in m)

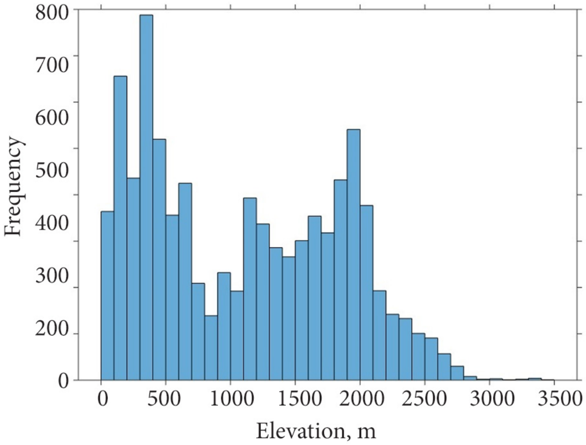

Figure 2. Elevation of gravity data over Kenya

1. Data

Gravity observations over Kenya can be traced back to 1899 (Searle, 1970). The gravity data over Kenya have been described by the following authors (Searle, 1970; Swain & Aftab Khan, 1977, 1978; Swain, 1979). More description of the gravity data can be found in Odera (2016). The current study used a catalogue of gravity data observed between 1955 and 1975 (Swain & Aftab Khan, 1977; Swain, 1979). A total of 8,690 gravity data covering the whole of Kenya are used. The accuracy of the gravity data is estimated as, ±0.1 ~ ±1 mGal (Swain & Aftab Khan, 1977). GPS/levelling data are only available in Nairobi City County and its environs (18 data points). Figure 1 shows the distribution of GPS/levelling and gravity data points in Kenya. The accuracy of GPS coordinates is given as, ±1 ~ ±2 cm for horizontal position and ±2 ~ ±4 cm for ellipsoidal height, while the allowable misclosure for the levelling network is $3\sqrt{K} \sim 8\sqrt{K}$ mm (where K is the levelling distance in km).

Elevations (on land) in Kenya range from 0 m in coastal areas to 5,199 m on top of Mt. Kenya (centrally located in Figure 1). The elevations of gravity data range from 1.7 to 3,437 m, with a mean and standard deviation of 1,106 and ±745 m respectively. Most gravity data (Figure 2) are in low areas (<1,000 m) with 4,125 data points (47.5%) followed by mid-elevation areas (1,000 ~ 2,000 m) with 3,420 data points (39.4%), high-elevation areas (2,000 ~ 3,000 m) with 1,134 (13.0%), and very-high-elevation areas (>3,000 m) with 11 data points (0.1%). The elevations for GPS/levelling data range from 1,534.5 to 2,144.2 m, covering a small portion of the country (Figure 1). This is a data limitation on the current research.

2. Methods

The assessment of the latest high-degree combined global gravity-field models is accomplished by comparing observed and GGMs implied free-air gravity anomalies and geoid undulations. The procedures used in the computations have already been described in Odera (2016) and discussed extensively by several authors (e.g. Heiskanen & Moritz, 1967; Rapp, 1971; Wichiencharoen, 1982; Torge, 2001). Only a summary, in the form of formulations, is given here with respect to observed free-air gravity anomaly (Equation (1)), GGM implied free-air gravity anomaly (Equation (2)) and GGM implied geoid undulation (Equation (3)). The observed geoid undulation is the difference between ellipsoidal height (normally obtained by GNSS) and orthometric height (normally obtained through precise levelling).

$$\Delta g_{FA} = g_{obs} + \delta g_{FA^S} + \delta g_{AC} - \gamma \,, \tag{1}$$

where Δg_{FA} is the observed free-air gravity anomaly, g_{obs} is the observed gravity, δg_{FA^S} is the second-order free-air reduction, δg_{AC} is a correction for the mass of the atmosphere and γ is the normal gravity based on a selected reference ellipsoid (GRS80 in this study).

$$\Delta g_{GGM} = \frac{GM}{r^2} \sum_{n=2}^{n_{\max}} \left(\frac{a_{ref}}{r} \right)^n (n-1) \times \sum_{m=0}^{n} (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\sin\phi); \tag{2}$$

$$N_{GGM} = N_o + C_t + [\frac{GM}{r\gamma} \sum_{n=2}^{n_{\max}} \left(\frac{a_{ref}}{r} \right)^n \times \sum_{m=0}^{n} (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\sin\phi)], \tag{3}$$

where Δg_{GGM} and N_{GGM} are the GGM implied gravity anomaly and geoid undulation respectively, N_o is the zero degree term (Heiskanen & Moritz, 1967), C_t is a conversion term used to convert height anomaly [third term (block brackets) in Equation (3)] into geoid undulation, *GM* is the product of the universal gravitational constant and mass of the Earth, ϕ and λ are geocentric latitude and longitude respectively, *r* is the geometric distance between the centre of the Earth and the computation point, a_{ref} is a scaling parameter associated with a particular GGM, $\overline{P}_{nm}(\sin\phi)$ are the fully normalised associated Legendre functions for degree *n* and order *m*, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are fully normalised spherical harmonic coefficients after reduction by the even zonal harmonics of the reference ellipsoid and $n_{\max}$ is the maximum degree of a GGM.

The use of residual terrain model (RTM) has become a standard method for estimating omission error in GGMs, hence augmenting the height and free-air gravity anomalies from GGMs. This procedure is normally applied to bridge the spectral gap between the GGMs and terrestrial data. However, a recent study by Gomez et al., 2017 has shown that the augmentation provided by RTM in high-resolution GGMs is very small. This validates the use of high degree GGMs (e.g. EGM2008) to cater for the omission error in the long-to-medium degree GGMs (e.g. GOCE models). An assessment of only high-degree GGMs to facilitate a selection of a candidate GGM for regional geoid modelling can therefore be achieved without the inclusion of the contribution of RTM. Hence, RTM is not used in the current study.

3. Results and discussion

The statistics of the differences between observed and GGMs implied geoid undulations and free-air gravity anomalies are shown in Tables 1 and 2 respectively. Table 1 shows that EIGEN-6C4 performs slightly better than EGM2008, followed by SGG-UGM-1 and GECO (in that order) in the recovery of geoid undulations over Nairobi City County and its environs. Table 2 shows that SGG-UGM-1 and EIGEN-6C4 perform at the same level in the recovery of free-air gravity anomalies over Kenya followed by EGM2008 and GECO in that order. It is clear from Tables 1 and 2 that any of the four high-degree GGMs can be used for the development of a precise geoid model in Kenya. However, EIGEN-6C4 is the preferred GGM, at least going by the independent test (Table 1), admittedly in a small area.

Table 1. Statistics of the differences between observed and GGMs implied geoid undulations over Nairobi area at 18 GPS/levelling data points (units are in cm)

GGM	Maximum	Minimum	Mean	SD
EGM2008	−46.68	−92.52	−72.94	11.62
EIGEN-6C4	−45.68	−91.02	−72.10	11.48
GECO	−35.85	−85.53	−65.42	12.51
SGG-UGM-1	−42.70	−89.44	−70.26	11.75

Table 2. Statistics of the differences between observed and GGMs implied free-air gravity anomalies over Kenya at 8,690 gravity data points (units are in mGal)

GGM	Maximum	Minimum	Mean	SD
EGM2008	64.90	−88.92	−1.04	10.11
EIGEN-6C4	68.02	−86.19	−1.08	10.03
GECO	67.00	−86.11	−0.69	10.19
SGG-UGM-1	66.16	−87.41	−0.95	10.00

The countrywide assessment of the four high-degree GGMs is only provided by the gravity data. The distribution of the differences between observed and GGMs implied free-air gravity anomalies are shown in Figure 3. Most of the free-air gravity anomalies can be recovered by the high-degree GGMs at ±10 mGal (79.2, 79.3, 79.3 and 79.6% for EGM2008, GECO, EIGEN-6C4 and SGG-UGM-1, respectively) and ±20 mGal (95.4, 95.5, 95.7 and 95.7% for EIGEN-6C4, EGM2008, SGG-UGM-1 and GECO, respectively). Clearly the slight improvement in gravity recovery by GECO, EIGEN-6C4 and SGG-UGM-1 is mainly due to the contribution of GOCE data in the long-to-medium wavelength components. The differences in free-air gravity anomalies from these GGMs are statistically insignificant, hence only EIGEN-6C4 (as a preferred GGM from the evaluations) is used in the subsequent results.

Figure 4 shows the differences between observed and EIGEN-6C4 implied free-air gravity anomalies over Kenya as a function of elevation, while Figure 5 shows a high correlation between observed and EIGEN-6C4 free-air gravity anomalies. Generally, larger differences in free-air gravity anomalies occur in high than low elevation areas (Figure 4). The correlation coefficient between observed

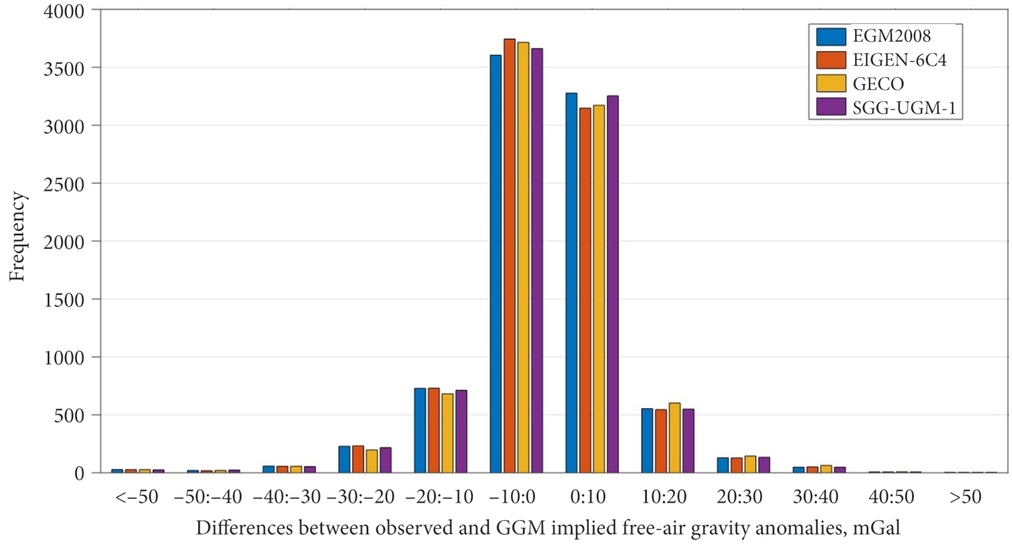

Figure 3. Differences between observed and GGMs implied gravity anomalies over Kenya at 8,690 gravity data points (units are in mGal)

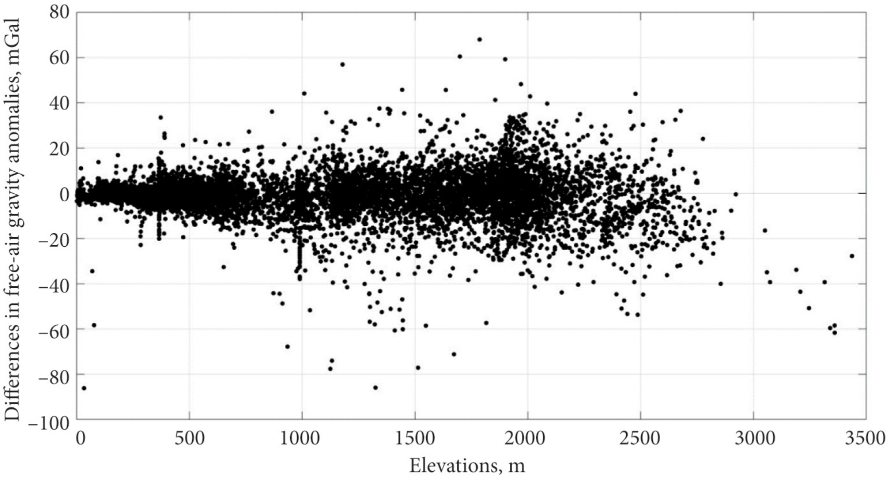

Figure 4. Differences between observed and EIGEN-6C4 implied free-air gravity anomalies over Kenya as a function of terrain elevation at 8,690 gravity data points

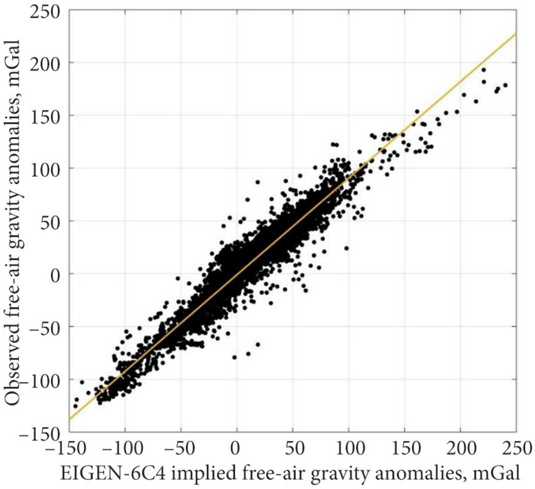

Figure 5. Correlation between observed and EIGEN-6C4 implied free-air gravity anomalies over Kenya at 8,690 gravity data points

and EIGEN-6C4 free-air gravity anomalies over Kenya is 0.97 (97%). This shows a good level of recovery of gravity-field over Kenya by the current high-degree GGMs (Figure 5). The spatial distribution of the differences between observed and EIGEN-6C4 implied free-air gravity anomalies over Kenya is shown in Figure 6. The significant differences are found in high areas and the rift valley. Smaller differences are found in low areas. This indicates that there is still a challenge in recovering gravity field in mountainous areas using the current high-degree combined GGMs.

The corrections to height anomalies to obtain geoid undulations computed by the conversion model (on a 2.5 arc-minute grid) are shown in Figure 7. These corrections range from −2.47 m on top of Mt. Kenya (5,199 m in central Kenya) to 0.01 m in low areas (eastern Kenya), with a mean value of −0.11 m and a standard deviation of ±0.18 m. These corrections are significant and cannot be ignored when determining geoid undulations over Kenya

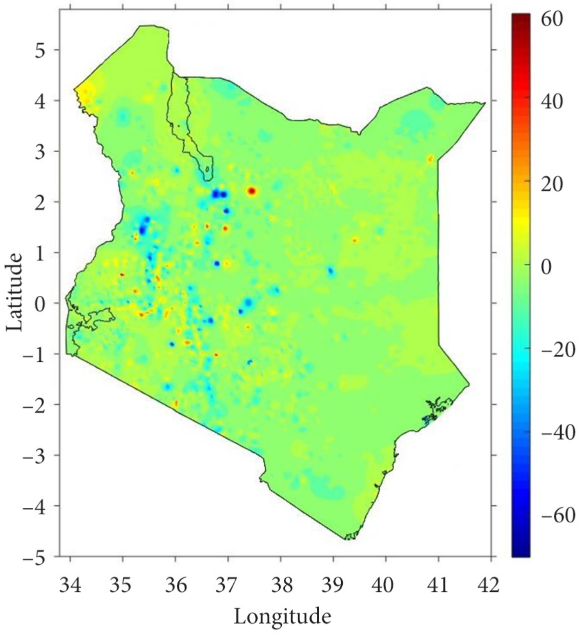

Figure 6. Spatial distribution of the differences between observed and EIGEN-6C4 implied free-air gravity anomalies over Kenya (units are in mGal)

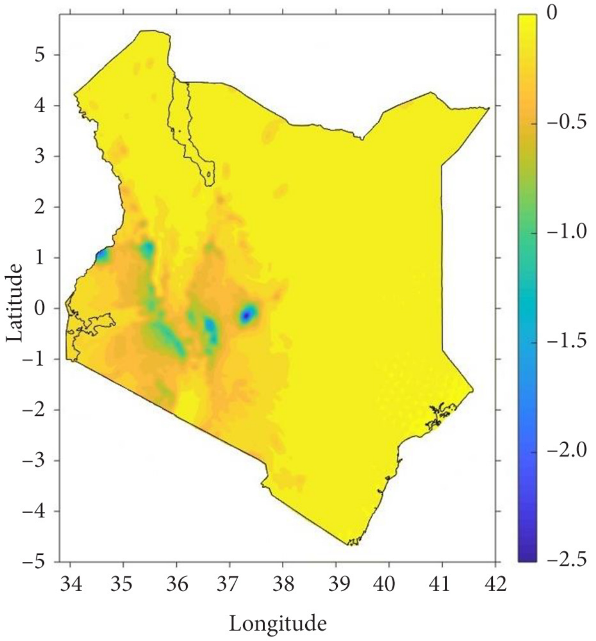

Figure 7. Spatial distribution of the corrections to height anomalies (to obtain geoid undulations) over Kenya (units are in m)

using global gravity-field models. These results demonstrate a known fact, that the separation between the geoid and the quasi-geoid is larger under mountainous areas, smaller in low areas and almost insignificant over the oceans. These corrections are applied to the height anomalies from EIGEN-6C4 on a 2.5 arc-minutes grid to obtain a GGM based geoid model over Kenya (Figure 8).

The geoid undulations over Kenya range from −37.3 m to −11.3 m, with a mean and standard deviation of −20.8 and ±5.7 m respectively. Larger geoid undulations occur under the mountainous areas in the western parts while smaller values occur in the low areas (eastern parts of Kenya). However, the separation between the reference ellipsoid and the geoid is larger in the eastern then western part of Kenya, in an absolute sense. It is important to note that the entire geoid model over Kenya is below the reference ellipsoid (GRS80) for a proper interpretation. A comparison between EIGEN-6C4-based gravimetric geoid model and the geoid model of the Eastern Africa region (Gachari & Olliver, 1998) would reveal more about the potential of the current GGMs in geoid modelling over Kenya. However, the Eastern Africa geoid model data was not available for the current study.

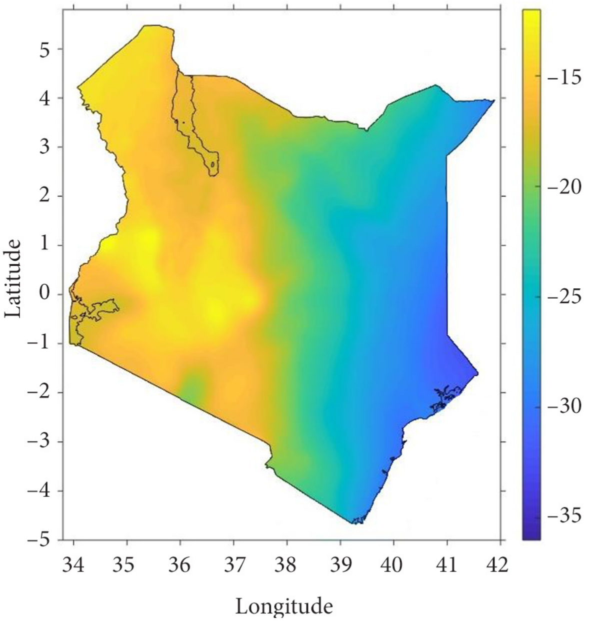

Figure 8. EIGEN-6C4-based gravimetric geoid model, with respect to GRS80, over Kenya (units are in m)

Conclusions

This study presents an initial evaluation of the recent high-degree global gravity-field models (EGM2008, EIGEN-6C4, GECO and SGG-UGM-1) over Kenya using geoid undulations and free-air gravity anomalies. The assessment of the GGMs based on geoid undulations is limited to Nairobi area, due to a lack of GPS/levelling data in other parts of Kenya. The assessment of free-air gravity anomalies is more comprehensive due to the availability of gravity data covering the whole country. A GGM-based geoid model for Kenya is developed from EIGEN-6C4 height anomalies and corrections from a conversion model, for the conversion of height anomalies to geoid undulations.

The results of the geoid undulation assessment (though on a small area and by only 18 GPS/levelling data points) reveal that EIGEN-6C4 performs slightly better than EGM2008, GECO and SGG-UGM-1. It (EIGEN-6C4) effectively recovers geoid undulations over Nairobi area at

±11.48 cm accuracy. All the models perform almost at the same level with exception of GECO, which performs relatively below the other models. The comparisons between observed and GGMs implied free-air gravity anomalies over Kenya, show, again that all the models perform at almost the same level with the best correspondence from SGG-UGM-1 (10.00 mGal), followed by EIGEN-6C4 (10.03 mGal), EGM2008 (10.11 mGal) and GECO (10.19 mGal). Strictly these results show that SGG-UGM-1 and EIGEN-6C4 can recover gravity-field over Kenya at the same level and better than EGM2008 and GECO. Clearly the contribution of GOCE data in improving gravity recovery in the long-to-medium wavelength components is discernable in the performance of EIGEN-6C4 and SGG-UGM-1. It is deduced that EIGEN-6C4 is currently the best high degree GGM for developing a precise geoid model over Kenya. It is expected that a more precise geoid model would be developed from a combination of EIGEN-6C4 and terrestrial gravity data over Kenya.

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