

INSPECTION PROGRAM DEVELOPMENT FOR FATIGUE CRACK GROWTH MODEL WITH TWO RANDOM PARAMETERS

Yuri Paramonov¹, Andrey Kuznetsov²

Aviation Institute of Riga Technical University, Riga, Latvia
E-mails: ¹rauprm@junik.lv; ²andreyk@hotmail.ru

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Yuri PARAMONOV, Prof Dr Habil

Date and place of birth: 28 March, 1938, Leningrad, Russia.

Education: Riga Aviation Engineering Military High School, Mechanical Engineering Diploma with Gold Medal (1960). Riga Civil Aviation Eng. Institute, Doctor Sc. Eng. Degree (1965). Latvian Academy of Sciences, High Doctor Degree in Technical Cybernetics (1974). Riga Aviation University and Latvian Academy of Science, Doctor Habilitus Degree in Engineering (1993).

Research interests: Reliability of technical systems; mathematical statistics; loads, structure and strength analysis of transport vehicle; American Biographical Institute.

Participation in international conferences: 15.

Publications: 228, including nine monographs and textbooks.

Honours, awards: Honoured Scientist of Latvian Soviet Socialist Republic (1983); order: Honour Decoration (1971); medals: For Valiant Labour (1970), Labour Veteran (1985); Nominations for an International Man of the Year for 1997/98 by the International Biographical Centre of Cambridge, and Great Minds of the 21st Century by American Biographical Institute.

Present position: professor of Aircraft Theory and Structure Department of Aviation Institute of Riga Technical University, Lomonosova Street, 1, Riga, LV-1019, Latvia, +(371) - 2255394, + (371) - 7089966.



Andrey KUZNETSOV, Dr

Date and place of birth: 30 January 1973; Riga, Latvia.

Education: Riga Aviation University (former Riga Civil Aviation Engineering Institute), Faculty of Radio Electronics and Computer Systems, Master Sc. Eng. (1995), Doctor Sc. Eng. Degree (2006); Chartered Institute of Bankers, Certificate in Bank Card Management (2000).

Research interests: Reliability of technical systems.

Participation in international conferences: 12.

Publications: 15.

Present position: director of Simourg, a business consultancy and software development company; lecturer at Aviation Institute of Riga Technical University.



Abstract. To keep the fatigue failure probability of an aircraft fleet at or below a certain level, an inspection program is appointed to discover fatigue cracks before they decrease the residual strength of some structurally significant item of the airframe lower than the level allowed by regulations. In this article, the p-set function for random vector, which, in fact, is a generalization of p-bound for random variable, and minimax approach to the problem of inspection number choice are used. It is supposed that the exponential approximation of a fatigue curve with two random parameters can be used in the interval when the fatigue curve becomes detectable and then grows to critical size. For estimation of distribution parameters, results of an approval test are used. A numerical example is given.

Keywords: p-set function, minimax approach, inspection program, and approval test.

1. Introduction

The development of an inspection program is necessary in order to provide reliability in a complex system. Examples of a solution to this problem and a lot of references can be found in books [1, 2, 3]. As a rule, a solution to this problem is provided under the condition that the cumulative distribution function (*cdf*) of time to failure is known. But really we should make some estimation of the *cdf* or at least the parameters of the *cdf* on the basis of processing the lifetime test result. A confidence interval is usually used for the estimation of the lifetime distribution parameter and then for the estimation of reliability. It is always very difficult to find a compromise between required reliability and confidence probability. But if we process some approval test data when we make some redesign of the tested system if some requirements are not met, then, as it will be shown later, it is possible to use the minimax approach, which provides required reliability independently of unknown parameters of lifetime distribution without using a confidence probability. For this purpose, the p-set function definition is used. Here we consider some example of p-set function application to the problem of development and control of an inspection program. We make the assumption that some structurally significant item (SSI), the failure of which is failure of the system being considered, are characterized by a random vector (r.v.) (T_d, T_c) , where T_c is critical lifetime (up to failure) and T_d is service time when some damage (fatigue crack) can be detected. So we have some time interval such that if in this interval some inspection will be fulfilled, then we can eliminate the failure of the SSI. We suppose also that the required operational life of the system is limited by the so-called specified life (SL), t_{SL} , when the system is discarded from service. In previous publications, we consider the case when in the equation of fatigue crack model there is only one random parameter [4–7]. This time we consider the case with two random parameters.

2. P-set function definition

P-set function for random vector is a special statistical decision function that, in fact, is a generalization of p-bound for a random variable, the definition of which was introduced much earlier. P-set function for random vector is defined in following way. Let Z and X be random vectors of m and n dimensions, and we suppose that the class $\{P_\theta, \theta \in \Omega\}$ is known. The probability distribution of the random vector $W = (Z, X)$ is assumed to belong to this class. Of the parameter θ , which labels the distribution, it is assumed known only that it lies in a certain set Ω , the parameter space. If $S_Z(x) = \bigcup_i S_{z,i}(x)$ is such set of disjoint sets of z values as function of x that :

$$\sup_\theta \sum_i P(Z \in S_{z,i}(X)) \leq p,$$

then the statistical decision function $S_z(x)$ is the p-set function for r.v. Z on the basis of the sample, $x=(x_1, \dots, x_n)$. Later on, the value x , the observation of the vector X , is interpreted as the result of some test or (sometimes it is more convenient) as the estimate $\hat{\theta} = \hat{\theta}(x)$ of the parameter θ . Z is interpreted as some random vector-characteristic of some SSI in service: for example, $Z = (T_d, T_c)$. For the development of the inspection program, the p-set function defines the sequence of inspection moments, which defines some set $S_z(x)$ of values of r.v. $Z = (T_d, T_c)$.

3. Development of inspection program

By processing the results of some special approval test (full-scale fatigue test of airframe, for instance), we can get the estimate $\hat{\theta}$ of parameter θ . The problem is to find (in a general case) a vector function $t(\hat{\theta})$, where $t = (t_1, t_2, \dots, t_n)$, t_i is the time of the i th inspection, $i=1, 2, \dots, n$, n is the inspection number $t_{n+1} = t_{SL}$ in such a way that the failure probability of the SSI under consideration

$$p_f(\theta, t) = \sum_{i=1}^n P(T_{i-1} \leq T_d < T_c < T_i),$$

does not exceed some small value ϵ :

$$\sup_\theta p_f(\theta, t) \leq \epsilon,$$

where T_1, \dots, T_n are random moments of inspections: r.v. $T = (T_1, \dots, T_n) = t(\hat{\theta})$; $T_0 = 0$; $T_{n+1} = t_{SL}$. This means that vector function $t(\hat{\theta})$ in fact defines some p-set function for vector (T_d, T_c) at $p = \epsilon$.

Usually we put $t_i = t_1 + d(i-1)$, $d = (t_{SL} - t_1)/n$, $i = 1, 2, \dots, n$. Then we need choose only t_1 and n . For the purpose of simplicity, we put $t_1 = d$ (in a general case t_1 can be chosen, for example, as parameter-free p-bound for T_c , or we can try to get the minimum of the expected value of n at fixed required reliability, etc) . Now the probability of failure will be the function of θ and n and we will denote it by $p_f(\theta, n)$. We suppose that $p_f(\theta, n)$ monotonically decreases when n increases (really this requirement is met only if n is large enough) and $\lim_{n \rightarrow \infty} p_f(\theta, n) = 0$ for all θ (Fig 4). Let $n(\theta, \epsilon)$ be the minimal inspection number n at which $p_f(\theta, n) \leq \epsilon$, where ϵ is some small value. But the true value of θ is not known. So $\hat{n} = n(\hat{\theta}, \epsilon)$ and $\hat{p}_f = p_f(\theta, \hat{n})$ are random variables. We suppose that we begin commercial production and operation only if some specific requirements are met. For example, the following requirements have to be met: 1) $\hat{n} \leq n_R$, 2) $\hat{t}_c > t_R, \dots$,

where n_R, t_R are some constants, \hat{t}_c is an estimate of the expected value of T_c . If these requirements are met, let us denote in a general case this event as $\hat{\theta} \in \Theta_0$, where $\Theta_0, \Theta_0 \subset \Omega$, is some part of parameter space. We suppose that if $\hat{\theta} \notin \Theta_0$ (estimate of required inspection number for some fixed ε exceeds some threshold n_R or estimate of expectation value of T_c , \hat{t}_c is too small in comparison with t_R), and then we redesign the SSI in such a way that probability of failure after this redesign will be equal to zero.

Let us define $\hat{p}_{f0} = \begin{cases} p_f(\theta, \hat{n}) & \text{if } \hat{\theta} \in \Theta_0, \\ 0 & \text{if } \hat{\theta} \notin \Theta_0. \end{cases}$

For this type of strategy the mean probability of fatigue failure $w(\theta, \varepsilon) = E_{\theta}(\hat{p}_{f0})$ is a function of θ and ε (Fig 1). If for limited t_{SL} it has a maximum, depending on ε then the choice of maximal value of $\varepsilon = \varepsilon^*$ for which $w^* = \max_{\theta} w(\theta, \varepsilon^*) \leq 1 - R$ and the strategy that defines the inspection number $n = n(\hat{\theta}, \varepsilon^*)$ is the strategy (decision function) for which the required reliability R is provided.

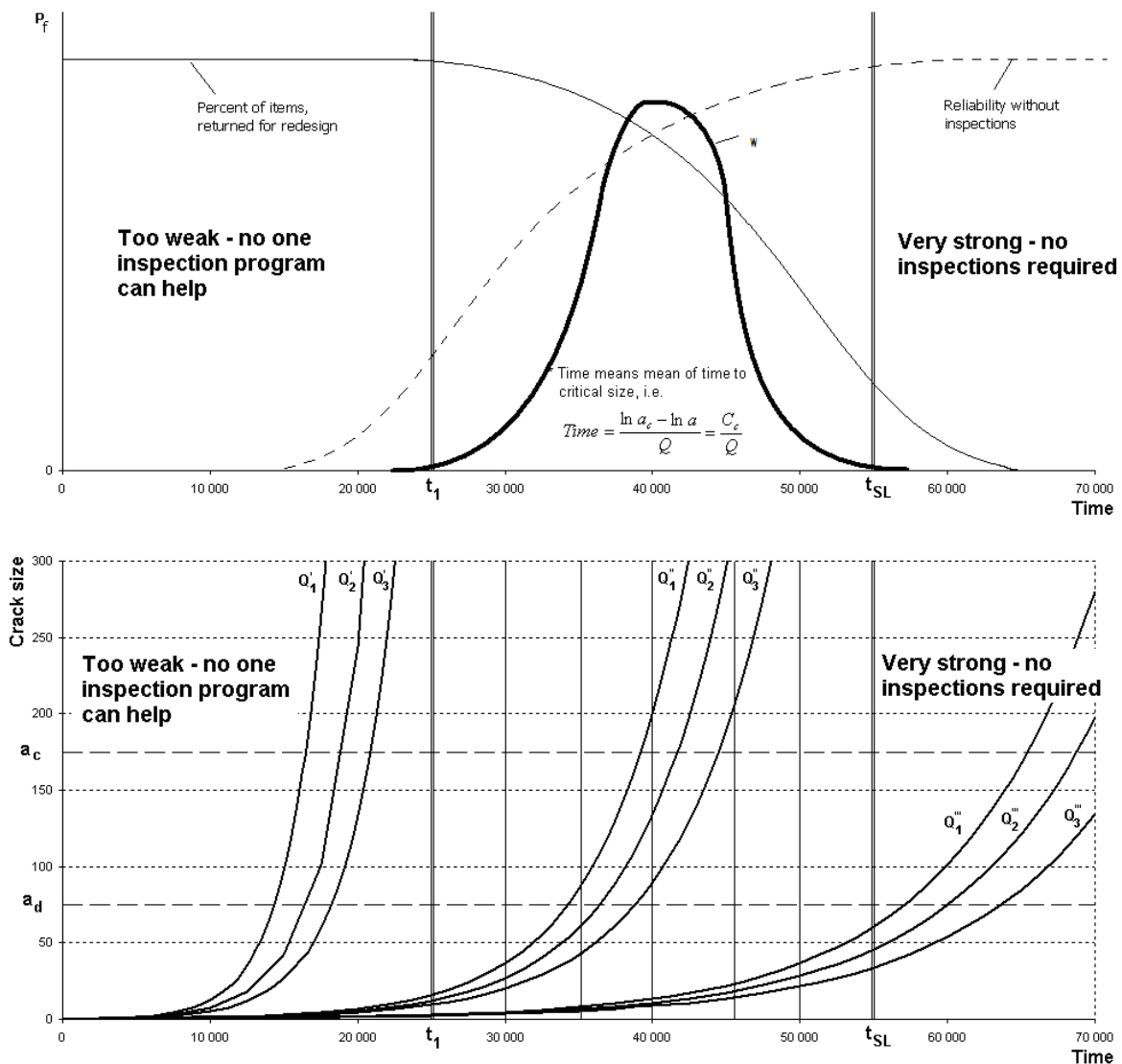


Fig 1. The value of $w = w(\theta, \varepsilon) = E_{\theta}(\hat{p}_{f0})$ as function of $E(T_c)$ for three design versions and corresponding random fatigue crack growth example sets

4. Exponential approximation of fatigue crack growth function

The numerical calculations will be based on exponential approximation of fatigue crack growth function when the size, $a(t)$, of a fatigue crack is described by the equation $a(t) = a(0)\exp(Qt)$. Despite its simplicity, this formula in the range of observation $[T_d, T_c]$, where T_d is the time when the crack becomes detectable and T_c is the time when the crack reaches its critical size, shows us rather comprehensible results. Then

$$T_d = (\ln a_d - \ln a_0) / Q = C_d / Q,$$

$$T_c = (\ln a_c - \ln a_0) / Q = C_c / Q,$$

where a_0 is $a(0)$, a_d is a crack size when the probability to discover it is equal to unit, and a_c is a crack size that corresponds to the maximum residual strength of an aircraft component allowed by special design regulation. We see that parameters C_c and C_d can be derived each be derived from the:

$$C_d = C_c - \delta, \text{ where } \delta = \ln a_c - \ln a_d = \ln \frac{a_c}{a_d},$$

so actually for the model of fatigue crack growth considered, the distribution of the r.v. (T_d, T_c) is defined only by two random variables (parameters): C_c and Q . They are the random fatigue crack growth model parameters (FCGMP).

Let us denote $X = \ln Q$ and $Y = \ln C_c = \ln(\ln(a_c/\alpha))$, so durability $T_c = C_c/Q$. From the analysis of the fatigue test data, it can be assumed that the logarithm of time required for the crack to grow to its critical size (logarithm of durability) is distributed normally: $\ln T_c \sim N(\mu_{\ln T_c}, \sigma_{\ln T_c}^2)$.

It comes from the additive property of the normal distribution that $\ln T_c$ could be normally distributed either if both $\ln C_c$ and $\ln Q$ ($C_c = \ln a_c - \ln \alpha$) are normally distributed (i.e. $X = \ln Q \sim N(\mu_X, \sigma_X^2)$, $Y = \ln C_c \sim N(\mu_Y, \sigma_Y^2)$), or if one of them is normally distributed while another one is a constant. In figure 2 these two cases are called one- and two-parametric models:

Let us denote the coefficient of correlation between X and Y by r .

For calculation of failure probability, $p_f(\theta, n)$, for the particular inspection program, we have to sum up all failure probabilities in all intervals as $p_f = \sum_{i=1}^{n+1} q_i$, where n represents the number of inspections,

$$q_i = P(t_{i-1} < T_d \leq T_c < t_i) = P\left(t_{i-1} < \frac{C_d}{Q} \leq \frac{C_c}{Q} < t_i\right)$$

$$= P(\ln C_c - \ln t_i \leq \ln Q < \ln(C_c - \delta) - \ln t_{i-1})$$

$$= \int_{\ln \delta}^{+\infty} (g_i^+(y)) d\Phi\left(\frac{y - \mu_Y}{\sigma_Y}\right),$$

where

$$g_i^+(y) = \max\left(0, \Phi\left(\frac{(\ln(e^y - \delta) - \ln t_{i-1}) - \mu_{X/Y}}{\sigma_{X/Y}}\right) - \Phi\left(\frac{(y - \ln t_i) - \mu_{X/Y}}{\sigma_{X/Y}}\right)\right)$$

$$\mu_{X/Y} = \mu_X + r \frac{\sigma_X}{\sigma_Y} (y - \mu_Y), \sigma_{X/Y} = \sigma_X \sqrt{1 - r^2},$$

$$\delta = \ln a_c - \ln a_d = \ln \frac{a_c}{a_d}.$$

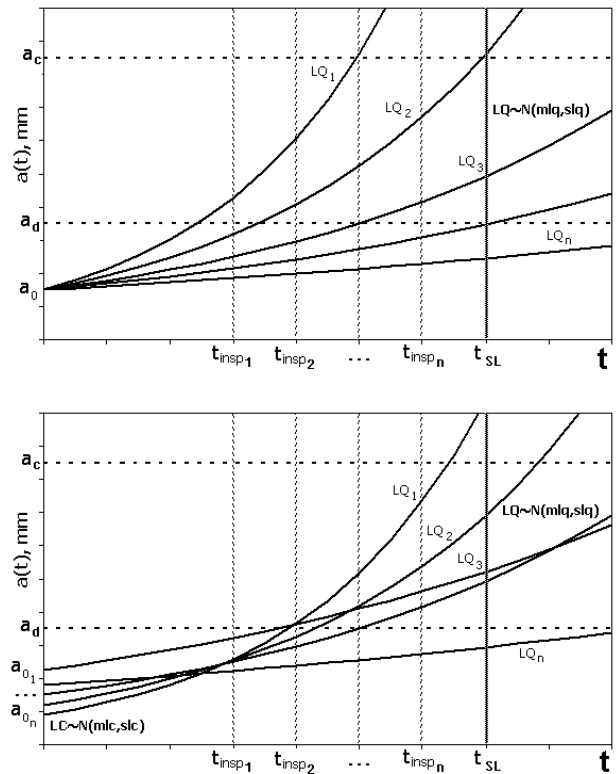


Fig 2. One- and two-parametric crack growth modelling (LQ=ln(Q))

In this paper we suppose that parameters σ_X, σ_Y and r depend on technology that does not change (for a new aircraft) and that these parameters can be estimated using information from previous designs. We suppose that they are fixed and are known values. Then unknown parameter, θ , have only two components: $\theta = (\mu_X, \mu_Y)$.

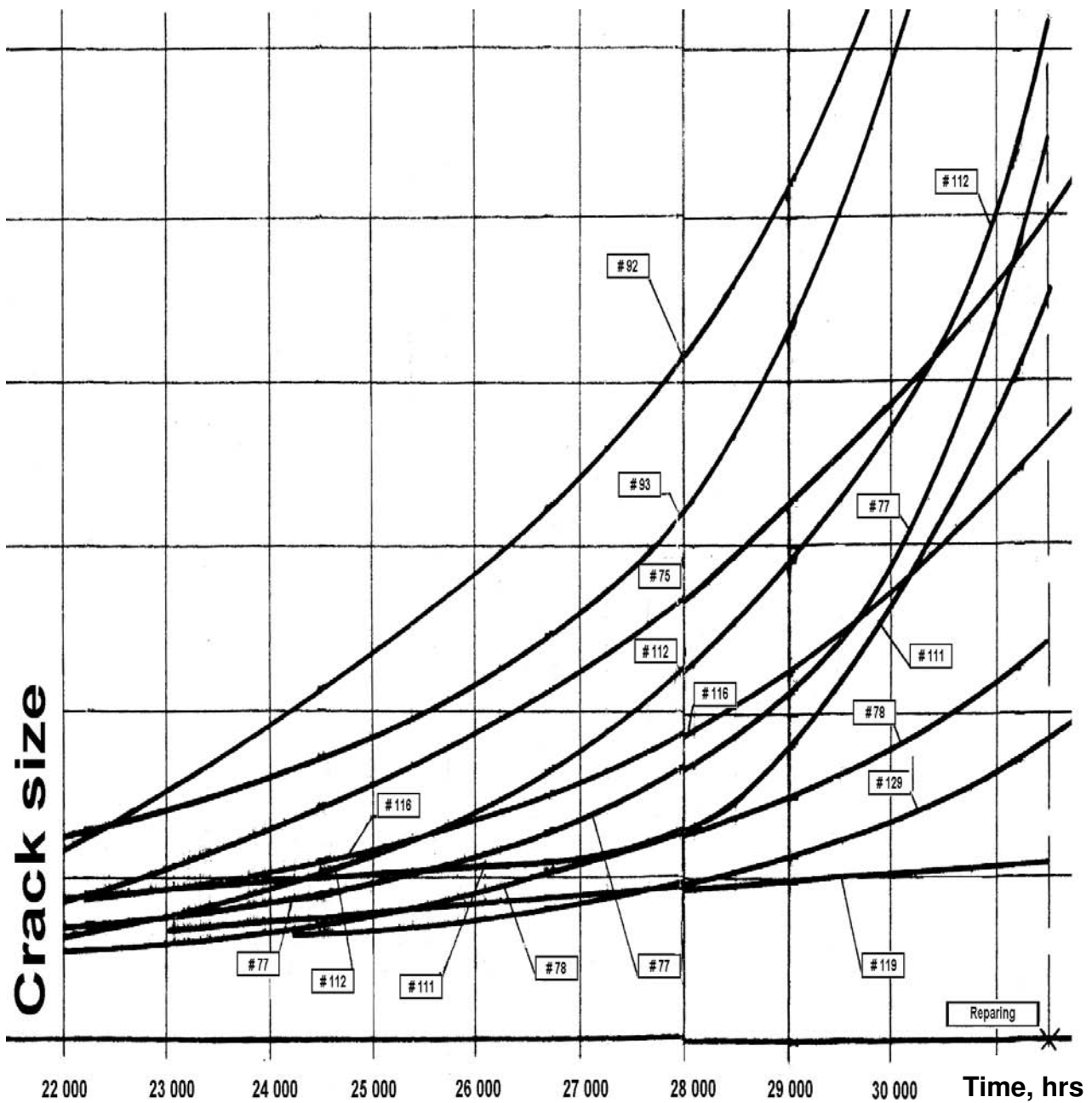
And for the considered decision-making procedure, the mean probability of fatigue failure $w(\theta, \varepsilon) = E_\theta(\hat{p}_{f0})$ is a function of μ_X, μ_Y and ε .

An example of FCGMP estimates using the observation of only one fatigue crack (it is typical information for the

program of inspection development) is shown in the upper part of figure 3. The linear (in logarithm scale: $\ln(a(t)) = Qt + \ln(a(0))$) regression analysis estimates of Q and $a(0)$ is shown. Using these estimates and known a_c , we can get estimates of μ_X and μ_Y : $\hat{\mu}_X$ is just equal to $\ln(Q)$ and $\hat{\mu}_Y$ is equal to $\ln(C_c)$, where $C_c = \ln(a_c) - \ln(a(0))$. To get estimates of σ_X, σ_Y and r , we made similar processing the observations of several cracks (see bottom part of figure 3), which, we assume, grow under the same stress level. In following calculation, the vector (σ_X, σ_Y, r) was considered some constant.

5. Numerical example

Let us demonstrate the approach described in previous sections on the numerical example. Suppose that we have only one fatigue crack observation and make an estimation of $\hat{\mu}_Y$ (Fig 3). And the vector (σ_X, σ_Y, r) is known. Around the point $(\hat{\mu}_X, \hat{\mu}_Y)$ we choose some area in plane $\{\mu_X, \mu_Y\}$. Using Monte Carlo modelling in other points of this area we make a calculation for some set of $\theta = (\mu_X, \mu_Y)$ in order to get the surface $w(\theta, \varepsilon) = E_{\theta}(\hat{p}_{f_0})$. In figures 4 and 5 are the results of modelling for $\varepsilon = 0.001$ and 0.005 are presented



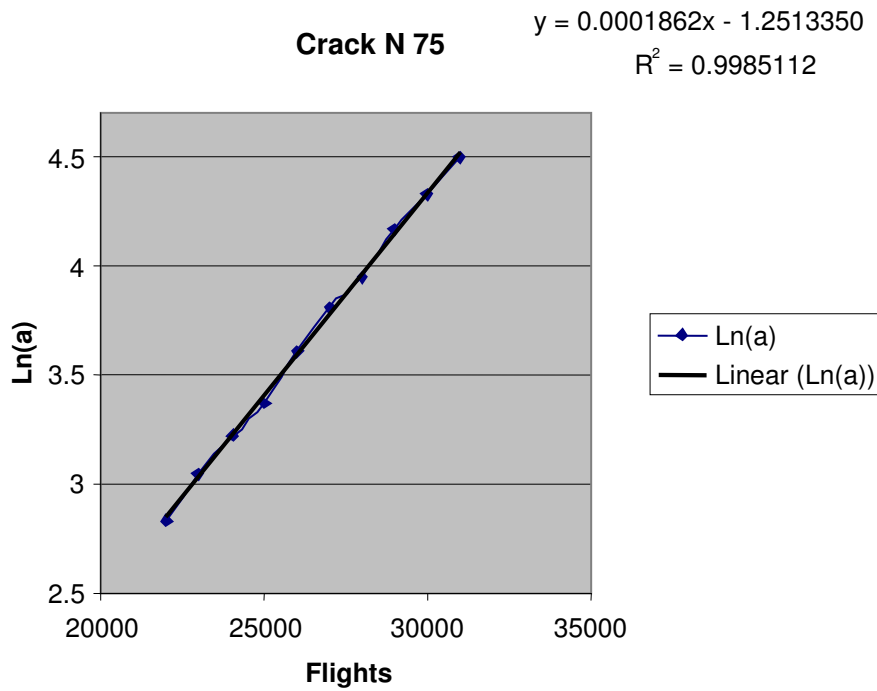


Fig 3. Example of FCGMP estimates and of some set of fatigue cracks under equal stress level

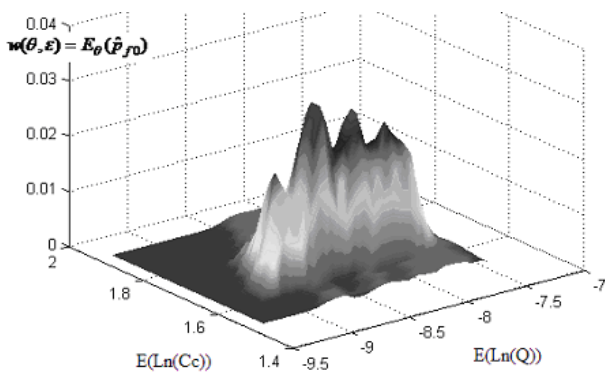


Fig 4. Numerical example for $\epsilon = 0.001$

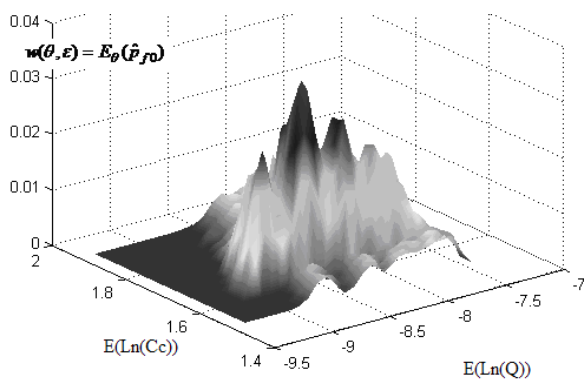


Fig 5. Numerical example for $\epsilon = 0.005$

(in this example, we use the inspection program with the special choice of t_1 and evenly distributed time moments

between t_1 and t_{SL} ; the time moment of the first inspection is defined as $t_1 = t_{SL} - 5 \cdot \theta_{1x}$; the detectable and critical crack sizes are $a_d = 20 \text{ mm}$, $a_c = 237.84 \text{ mm}$). For these examples we assume that only one full-scale test was performed and we have data on just one single crack growth (crack #75: $\ln Q = -8.588527$, $\ln C_c = 1.905525$). Let us say that we have to ensure the probability of failure not exceeding 0.0326 and that we will return for redesign all projects when required number of inspections exceeds $n_R = 5$. If we perform modelling using various values of failure probability ϵ , we will get a set of “surfaces” $w(\theta, \epsilon) = E_\theta(\hat{p}_{f0})$.

The maximum values of the function $w(\theta, \epsilon)$, $w^*(\epsilon) = \max_\theta w(\theta, \epsilon)$, are equal to 0.030990 and 0.033874 for $\epsilon = 0.001$ and 0.005 correspondingly. A similar calculation gives $w^*(\epsilon) = \max_\theta w(\theta, \epsilon) = 0.032593$ for $\epsilon = 0.003$.

The complex form of the function $w(\theta, \epsilon)$ is defined by the fact that $p_f(\theta, n)$ might be the non-monotonous function of n . For relatively small n , $p_f(\theta, n)$ can grow with the increase of n . The reason of such a “strange” effect comes from the relocation of the inspection time with the change in n . The example pictured in figure 6 demonstrates how a crack, discoverable with a single-inspection program, is missed if an inspection program with two inspections is applied. The function $w^*(\epsilon)$ is shown in figure 7.

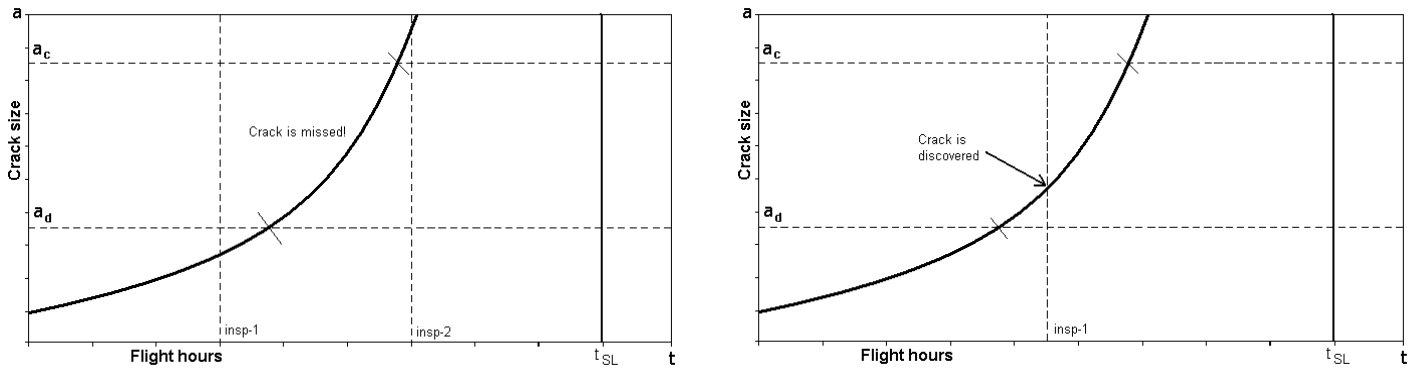


Fig 6. Demonstration of non-monotonous nature of $p_f(\theta, n)$

In our example we see that to ensure the probability of failure not exceeding $w^* = 0.0326$ at the choice of n^* , the required number of inspections for our inspection program (using formula: $n = \min(n: p_f(\theta, n) < \varepsilon)$), we have to use the value $\varepsilon = \varepsilon^* = 0.003$ (it is worth mentioning that w^* is ten times higher than ε^* !) (Fig 8).

The required number of inspections in our example is $n^* = 5$ (the same data gives the required number of inspections $n = 4$ for $\varepsilon = 0.0326$).

6. Conclusion

This procedure for the development of an inspection program is offered for the case when the exponential model of fatigue crack growth has two random parameters. The p-set function and minimax approach are offered for the choice of inspection number using the results of a full-scale fatigue test on an airframe. It is shown that if instead of using unknown parameters of the exponential fatigue crack growth model, we can use the estimates of a parameter (processing only one fatigue crack observation), then the real probability can be 10 times more than one that was used for the inspection number calculation. For the case of approval fatigue test, when we redesign the tested airframe if some requirements are not met, the minimax statistical decision functions allow us to find a decision that provides the required reliability of airframe in operation.

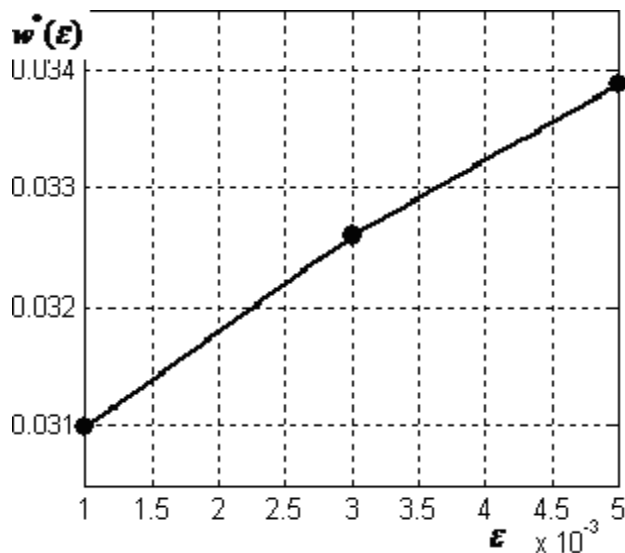


Fig 7. Numerical example: the function $w^*(\varepsilon)$

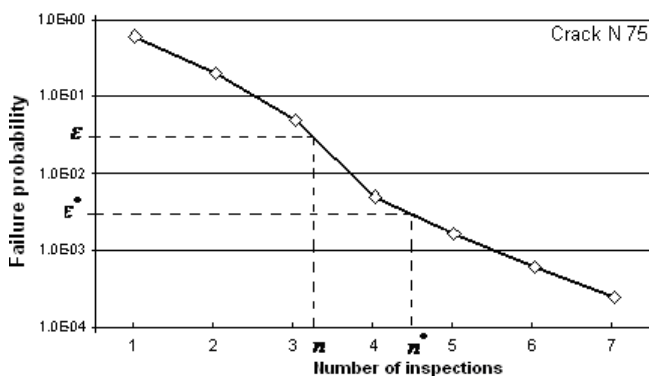


Fig 8. Numerical example: determining required number of inspections

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APŽIŪRŲ PROGRAMOS KŪRIMAS NUOVARGIO PLYŠIO AUGIMO MODELIIUI SU DVIEM ATSITIKTINIAIS PARAMETRAIS

Yu. Paramonov, A. Kuznetsov

S a n t r a u k a

Šiame tyrime nagrinėtas apžiūrų, skirtų surasti nuovargio įtrūkimus jėginiuose elementuose iki liekamojo stiprumo sumažėjimo žemiau leistinos ribos, programos planavimas. Čia apžiūrų skaičiui nustatyti buvo naudojamas mini-maksimalus statistinis sprendinys ir atsitiktinio vektoriaus p-aibės sąvoka, kuri yra atsitiktinio vektoriaus p-ribos apibendrinta sąvoka. Taikyta prielaida, kad nuovargio įtrūkimo didėjimo kreivę galima aproksimuoti eksponentiškai laiko intervale nuo to momento, kai plyšys tampa matomas ir iki kritinio dydžio. Parametrų pasiskirstymo įvertinimui naudoti bandymo rezultatai.

Daroma prielaida, kad jei bandymo rezultatai yra nepatenkinami, tuomet turi būti ruošiamas naujas, labai pagerintas bandomojo gaminio projektas. Pateikti ir skaitiniai pavyzdžiai.

Reikšminiai žodžiai: p-aibės sąvoka, mini-maksimalus sprendinys, apžiūrų programa, aprobavimo testas.